

STATE UNIVERSITY OF NEW YORK AT STONY BROOK  
DEPARTMENT OF PHYSICS AND ASTRONOMY

**Part I.**

**Tuesday, 19 January 1999 – Day 1**

**Comprehensive Examination in Classical Mechanics and Special Relativity  
and in Electromagnetism and Optics**

**General instructions:** In each of the two areas, do two of the three problems. Each problem should take about  $\frac{3}{4}$  hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

**Classical Mechanics and Special Relativity**

Three problems, work any two.

CM I. (Tolpygo)

A pendulum (mass  $m$  attached to the end of an infinitely thin, massless rod of length  $l$ ) is immersed in a viscous medium (friction coefficient  $\mu$ ) that provides a friction force proportional to the velocity of the mass. The acceleration of gravity is  $g$ . The pendulum is rigidly attached to a massless pulley of radius  $R$  as shown in the figure. The pendulum was at rest until at  $t = 0$ , a constant force  $f$  was applied to a massless rope wound on the pulley.

- a. (3 points) Describe qualitatively the possible types of motion of the pendulum at  $t > 0$ . Find the critical value  $f_c$  of the force separating the possible types of motion.
- b. (3 points) Derive a differential equation describing the motion of the pendulum.
- c. (6 points) Find the frequency and the decay time of small oscillations of the pendulum around the equilibrium position at  $f < f_c$ .
- d. (3 points) Find the value of the friction coefficient at which this motion becomes overdamped (non-oscillatory).
- e. (5 points) Find the frequency of the motion of the pendulum at  $f \gg f_c$ .

CM II. (Pantuev)

A massless spring has spring constant  $k$  and length  $l$  when it is not compressed. At each end of the spring is a pointlike mass  $m$ . Let the system vibrate and rotate with angular momentum  $L$  and total energy  $E$ .

- a. Find the frequency of oscillations for the given motion.
- b. Write the equation that defines the maximum length of the spring for the given motion.
- c. Solve this equation approximately, assuming that the deviation from equilibrium is small.

CM III. (Goldhaber)

In the following, ignore possible curvature of spacetime. At time  $t = 0$  in a frame comoving with the earth, a distant source at  $D = 10^{10}$  light year with a recession velocity  $v = 1.8 \times 10^8$  m/s emits a pulse of green light (wavelength  $\lambda = 550$  nm).

- a. Deduce the time of arrival of the pulse at earth.
- b. What is the shift in wavelength  $\Delta\lambda$  of the arriving light as seen by an observer on earth?
- c. If at  $t = 0$  also a light pulse is sent from earth to the source, how long does it take to get there?
- d. In a frame comoving with the distant source, how long was the travel time of the pulse in (c)?

## Electricity and Magnetism and Optics

Three problems, work any two.

E&M I. (Kahn)

Two concentric spherical surfaces have radii  $R_1 = 1$  m and  $R_2 = 2$  m. The region between  $R_1$  and  $R_2$  is charge-free.

- a. (5 points) If the potential at all points on the inner surface is maintained at 80 volts and the potential at all points on the outer surface is maintained at 700 volts, find an expression for the potential  $V(r)$  at any radius between  $R_1$  and  $R_2$ .
- b. (15 points) Suppose that the inner surface is held at 80 volts but that the potential on the outer surface is  $700(1 - \cos \theta)$  volts, where  $\theta$  is the polar angle ( $\theta = 0$  on the positive  $z$ -axis). Find an expression for the potential  $V(r, \theta)$  for points between  $R_1$  and  $R_2$ .

E&M II. (Smith)

A long, straight, heated wire of radius  $a$  is surrounded by a concentric cylinder of radius  $b$ . The cylinder is maintained at a potential  $+V_0$  relative to the wire. The length of the wire and cylinder is  $l$ .

- a. (5 points) If a single electron is emitted from the wire with negligible initial speed, find the speed of the electron  $v(r)$  when the electron is a distance  $r$  ( $a < r < b$ ) from the center of the wire.
- b. (15 points) Suppose many such electrons are emitted continually from the wire, so that an average electron current  $I_0$  is maintained between the wire and the cylinder. Find the potential  $V(r)$  for any distance  $r$  ( $a < r < b$ ), assuming that the effects of space charge (i.e., the electrons in transit) can NOT be neglected. What is the functional relationship between  $V_0$  and  $I_0$ ?

Hint: In cylindrical coordinates Poisson's equation is  $\frac{d^2V}{dr^2} + \frac{1}{r} \frac{dV}{dr} = -4\pi\rho$ .

E&M III. (Metcalf)

A single-frequency laser beam of wavelength  $\lambda = 600$  nm (in air) shines directly into a power meter that reads 1.00 mW.

- a. (2 points) A flat, lossless mirror of 99% reflectivity is placed between the laser and the meter with its normal parallel to the laser beam. What does the power meter read?
- b. (5 points) A second identical mirror is similarly placed 6 cm behind the first mirror (between the first mirror and the detector), to form a Fabry-Perot interferometer. Now the power meter reads 1.00 mW again. Explain why. How can the 99% of the light that is reflected by the first mirror in (a) now be redirected to the power meter in this second configuration?
- c. (7 points) Now the second mirror is moved slowly toward the first one, and the meter reading varies with the position. Draw a graph of the power *vs.* displacement of the second mirror over the range of  $2 \mu\text{m}$ . Label both axes quantitatively.
- d. (4 points) The spectral features that appear in the graph of part (c) have a certain width, even if the laser is truly “single frequency”. Estimate that width and justify your answer.
- e. (2 points) How does the graph of part (c) change if the chamber holding the interferometer is evacuated? Draw a new figure and label the axes. You have to provide your own estimate of the index of refraction of air.