

STATE UNIVERSITY OF NEW YORK AT STONY BROOK
DEPARTMENT OF PHYSICS AND ASTRONOMY

Part II.

Thursday, 21 January 1999 – Day 3

Comprehensive Examination in “Experiment” and “Breadth”

General instructions: Twelve problems are given. You should do any four, subject to the constraint that you should answer **no more than three** from “experiment” and **no more than three** from “breadth” (that is, not all four problems can be chosen from the same category). Each problem should take about $\frac{3}{4}$ hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem’s author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, and, with the proctor’s approval, a foreign language dictionary. None of these materials may be shared between students.

“Experiment”

Experiment I. (Koch)

Do any 4 of the following 5 parts. Each of the 4 will be worth 5 points.

How can you determine experimentally to the stated level of accuracy the following

- a. to a factor of 2 or better the next time you pump it down, the pressure inside a room temperature vacuum vessel in which the density of gas will be approximately 10^9 molecules/cm³? (Numbers that may be useful: atmospheric pressure is 1.013×10^5 Pa = 760 Torr; Boltzmann’s constant is $k_b = 1.38 \times 10^{-23}$ J/K.)
- b. to 30% or better the microwave power in watts that a microwave oven actually delivers (at the hundreds of watts level) to an (absorbing) load placed inside?
- c. to 10% or better the characteristic impedance Z_{char} (in ohms) of a 2 meter long piece of coaxial cable that has BNC connectors at each end?
- d. to 5% or better the input resistance (real part of the input impedance) R_{in} (in ohms) of an oscilloscope at a frequency of 100 kHz?
- e. to 5% or better at $\lambda = 632.8$ nm the (real part of the) index of refraction n of a polished, rectilinear block of black (opaque) volcanic glass sitting in air?

Experiment II. (Stephens)

You have several disks of doped silicon, 0.5 mm thick, 3 cm diameter. You want to measure the dielectric constant and conductivity of each one. The dielectric constant of Si is about 20. You cover the flat surfaces with metal (contact resistance is negligible), and you connect this “capacitor” into a circuit in series with a 100 k Ω reference resistor. You have a 100 kHz, 1.00 V voltage source and a lockin amplifier. The lockin can measure the in-phase and out-of-phase signals with an accuracy of 0.1% relative to the total signal. In the lowest sensitivity setting the lockin can measure 10 V; in highest sensitivity setting it can detect 0.1 μ V. It has an input impedance of 100 M Ω .

- a. How would you set up the measurement? (Provide a schematic diagram of connections to: lockin input, reference channel, sample, etc.) How would you evaluate the resistivity ρ and dielectric constant κ from the measured data?
- b. What is the range of conductivities that can be measured with this setup? (Order of magnitude estimates are OK. We are more interested in the reasoning than in precise numerical values.)
- c. If one of your samples has a resistivity of $10^4 \Omega\text{cm}$, how accurately can you determine its dielectric constant? (Likewise, valid reasoning is more important than precise numerics.)

(Of course everybody knows that the permittivity of free space is 8.85×10^{-12} F/m. If you cannot come up with the numerical factors in any of the formulas you need, make plausible guesses.)

Experiment III. (LaFosse)

In a typical low/medium energy nuclear physics experiment, a beam of nuclei is aimed at a target to induce a nuclear reaction. Information concerning nuclear forces and nuclear structure is obtained by studying the charged particles (protons and α particles), neutrons and γ rays emitted by the excited compound system. Typical energies are: protons and α particles: 1–10 MeV; γ rays: 100 keV to 5 MeV; neutrons: 1–10 MeV. A typical detector system might consist of several solid-state, high-purity Ge detectors for the detection of γ rays, several inorganic scintillator detectors (CsI, NaI) for detecting charged particles, and several plastic scintillators for the detection of neutrons. Finally, the residual nucleus (i.e. the compound nucleus after the emission of particles) might be identified in an ionization chamber. (For the sake of this problem, you may assume the residual nucleus will have kinetic energy on the order of 80 MeV, have mass $A \sim 80$, and atomic number $Z=35$.)

Describe how the above mentioned detectors detect radiation(s). Specifically, describe a) how solid state detectors work in general, and specifically how they detect γ rays; b) how scintillator detectors work and detect charged particles; c) how neutrons are detected in scintillator detectors; and d) how ionization chambers work, and how they are able to distinguish between nuclei having different values of Z (atomic number). Be sure also to discuss the rationale for choosing each specific type of detector for the specific type of radiation. (E.g., discuss why high-purity Ge detectors are used to detect γ rays in these experiments, but scintillators and ionization chambers are not.)

Experiment IV. (Rijssenbeek)

Consider the production of Z bosons in the CERN Large Electron-Positron collider (LEP), which straddles the Swiss-French border near Geneva. These neutral vector bosons have a large mass of $M_Z = 91,187 \pm 7 \text{ MeV}/c^2$ and a total width $\Gamma_Z = 2,490 \pm 7 \text{ MeV}/c^2$. The LEP collider has counter-rotating electron and positron beams, which are focussed at the four interaction points, where large detectors measure the products of the e^+e^- collisions. The total energy in the center of mass system, which is also the laboratory system, is \sqrt{s} , with the Mandelstam variable $s \equiv (p_{e^+} + p_{e^-})^2$; i.e., the squared sum of the positron and electron four-vectors. The luminosity of the collider, \mathcal{L} , is the rate of interactions observed in the collision region per unit of cross section, and it depends on the machine parameters; for LEP: $\mathcal{L} \simeq 2.4 \times 10^{31} \text{ s}^{-1} \text{ cm}^{-2}$.

- a. (3 points) Explain why a large radius (about 4 km, and hence a low bending power) collider ring is needed to efficiently accelerate and collide electrons and positrons. What limits the maximum energy of an electron-positron collider ring?
- b. (2 points) The production cross section of muon pairs near the Z resonance, calculated to first order, equals:

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = 12\pi \left(\frac{s}{M_Z^2} \right) \frac{\Gamma_{ee}\Gamma_{\mu\mu}}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2},$$

where M_Z and Γ_Z denote the mass and the total width of the Z boson, and where Γ_{ee} and $\Gamma_{\mu\mu}$ are the partial widths of the Z boson into (initial state) electrons and (final state) muons: $\Gamma_{ee} = \Gamma_{\mu\mu} = 83.8 \pm 0.3 \text{ MeV}$. The above formula uses $\hbar = c = 1$;

Show that this cross section at the Z peak equals about 2.0 nb (1 barn $\equiv 10^{-24} \text{ cm}^2 = 100 \text{ fm}^2$).

- c. (2 points) Sketch the first-order Feynman diagram(s) that describe the process above.
- d. (3 points) Based on the previous parts, calculate the total cross section $\sigma_{tot}(e^+e^- \rightarrow X)$ at the Z resonance peak, and calculate the total rate of interactions at a LEP interaction region.
- e. (10 points) Describe an effective identification method for each of the following objects at the energy $E \simeq M_Z c^2/2$:
 - i. electron
 - ii. photon
 - iii. muon
 - iv. neutrino
 - v. b -quark

Experiment V. (Walter)

The binary star system AR Lacertae consists of a K0 giant and a G2 subgiant in a circular orbit with a 1.983 day period. The orbital velocities of the stars are nearly equal, at 116 and 115 km s⁻¹ for the K0 and G2 stars, respectively. We see the system nearly edge-on, so it undergoes total eclipses (use $i = 90^\circ$ throughout). The duration of the total eclipse is 2.2 hours, and the partial phase (first contact to start of totality) lasts 2.5 hours.

- a. (10 points) What are the masses and radii of the stars?
- b. (6 points) The observed brightness (integrated over all wavelengths) at primary minimum is 69% of the out-of-eclipse brightness, and 80% at secondary minimum. Treat each stellar disc as a blackbody with uniform observed surface brightness. The G2 subgiant has an effective temperature T_{eff} of 5800K. What is the temperature of the K0 giant?
- c. (4 points) Is the difference in evolutionary status between the stars in agreement with the mass difference implied by the difference in orbital velocities? (Explain, but limit answer to 1 paragraph)

Experiment VI. (Solomon)

In this problem, we examine the Dark Matter problem.

- a.** (5 points) There is substantial evidence for the existence of Dark Matter in spiral galaxies, particularly in their outer parts. State what precisely we mean by the Dark Matter problem.
- b.** (15 points) The observations required to demonstrate the Dark Matter problem in spiral galaxies can be carried out at optical wavelengths or at a combination of optical and radio wavelengths. Choose a particular sensible wavelength, or wavelengths, for this observational demonstration and briefly describe the observations. In particular, specify *(i)* the wavelengths chosen, *(ii)* the types of instruments and/or telescopes used, *(iii)* the type(s) of data to be obtained with them, and *(iv)* the likely results.

“Breadth”

Breadth I. (Bergeman)

Positronium consists of an electron plus a positron. It has been carefully studied for various tests of quantum electrodynamics. Following the Russell-Saunders (LS) coupling scheme used for “normal” light atoms, we may label levels down to the level of fine-structure with the notation $n^{2S+1}L_J$, where n is the principal quantum number, S is the total spin angular momentum quantum number, L is the total orbital angular momentum quantum number, and J takes on the value(s) permitted by quantum mechanics for coupling S and L .

- a. Discuss how positronium resembles (and how it differs) from the hydrogen atom.
- b. In view of (a), give values for binding energies of the $n = 1$ and 2 levels of positronium, accurate to 0.1 eV.

The ground 1^1S_0 state of positronium has a lifetime of about 0.12 ns and decays into an even number of gamma ray photons. The 1^3S_1 level, just 203 GHz above, has a lifetime of about 142 ns and decays into an odd number (greater than 1) of gamma ray photons. Consider the following:

- c. The $2^2S_{1/2}$ state of **hydrogen** is metastable (it has a lifetime of about 1/7 second). Why is it metastable? How can it decay?
- d. Would you expect the positronium 2^3S_1 state to be metastable? What do you estimate to be the dominant decay process, and what is its rate?

Breadth II. (Aleiner)

Below are shown two views of the carrier density versus reciprocal temperature for a lightly n -doped semiconductor. The slopes of the straight sections at small and large $1/T$ are -3.7 K and -0.032 K. Slopes should be interpreted as $\Delta(\log_{10} n)/\Delta(1000/T)$, with n measured in cm^{-3} .

- a.** How might this have been measured? What experiment tells that the material is n -doped rather than p -doped? What is the donor concentration n_D ?
- b.** Interpret physically each of the three straight sections on the graphs.
- c.** In order to get formulas for the carrier density as a function of n_D and other parameters, it is necessary to figure out the value of the chemical potential μ at each temperature. Sketch or explain how μ varies with $1/T$ or T in the three regimes mentioned above.
- d.** Extract from the graphs and give numerical values for two important parameters of this semiconductor: the gap width E_g and the donor binding energy ϵ_D .

Breadth III. (Prakash)

The mass of an atomic nucleus with Z protons and N neutrons (mass number $A = N + Z$) is given by $M(Z, A) = ZM_H + (A - Z)m_n - B$, where M_H is the mass of the neutral hydrogen atom, m_n is the mass of the neutron, and B is the binding energy of the nucleus. In the liquid-drop model of the nucleus, B is expressed as

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + \text{small terms},$$

where a_v (the bulk volume energy coefficient) $\simeq 16$ MeV, a_s (the surface energy coefficient) $\simeq 18$ MeV, a_c (the Coulomb energy coefficient) $\simeq 0.7$ MeV, and a_a (the bulk asymmetry energy coefficient) $\simeq 30$ MeV.

- a.** (8 points) By treating the N neutrons and Z protons in a nucleus as ideal, nonrelativistic Fermi gases, derive an expression for the total energy of the system. By considering deviations from the case $N = Z$, derive the asymmetry term a_a in terms of the Fermi energy of the $N = Z$ system. Assuming that the Fermi momentum of the $N = Z$ system is $p_F = 260$ MeV/ c , make a numerical estimate of a_a . Contrast your answer with the empirical value given above along with a physical interpretation.
- b.** (8 points) Using the mass formula above, derive an expression for the energy released in the symmetrical fission of a nucleus of charge Z and mass number A . Give a numerical estimate for the symmetric fission of ${}^{236}_{92}\text{U}$.
- c.** (4 points) On the $Z - N$ plane, sketch the region of nuclei that are unstable toward fission.

Breadth IV. (McCarthy)

Consider a two-component model for neutrino mixing

$$|\nu_\mu\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

$$|\nu_\tau\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

where the μ and τ lepton-flavor eigenstates are expressed in terms of mass eigenstates via the mixing angle θ . Consider a μ -neutrino which is produced at time $t = 0$ with momentum p and energy E where $E \gg m_1$ and $E \gg m_2$. (The quantities p , m_1 , and m_2 are expressed in energy units.)

a. Show that after a time t this state becomes

$$|\nu_\mu(t)\rangle = \exp(-i\left(\frac{pt}{\hbar}\right))\left(\exp(-i\left(\frac{m_1^2 t}{2p\hbar}\right))\cos\theta|\nu_1\rangle + \exp(-i\left(\frac{m_2^2 t}{2p\hbar}\right))\sin\theta|\nu_2\rangle\right)$$

b. Show that the probability that this μ -neutrino will “disappear” (be observed as a τ -neutrino) at time t is given by

$$|\langle\nu_\tau(0)|\nu_\mu(t)\rangle|^2 = \sin^2 2\theta \sin^2 \left[\frac{\Delta m^2 L}{4E\hbar} \right]$$

where $\Delta m^2 = m_2^2 - m_1^2$ and L is the distance traveled at time t .

c. Briefly discuss the results of one recent experimental search for neutrino oscillations.

Breadth V. (Lattimer)

- a. (12 points) White dwarfs (i.e., cold objects supported by electron degeneracy pressure against gravity) have a maximum mass, the so-called Chandrasekhar mass, M_{Ch} . Show that there is a maximum mass and express it in terms of fundamental constants. (You may use dimensional analysis and neglect factors of order unity.)
- b. (3 points) How does the reasoning change for a cold object supported by nucleon degeneracy (approximately a neutron star)?
- c. (5 points) Why must there be a minimum mass for neutron stars?

Breadth VI. (Lanzetta)

In this problem, we investigate a simple theory of gravitational lenses.

- a. (8 points) Consider a non-relativistic particle that travels with speed v past a spherically symmetric galaxy for which $M(r)$ is the mass enclosed within radius r . Assume that the velocity v of the particle is sufficiently large that the deflection of the trajectory due to the gravitational force from the galaxy is small, and let b be the distance of closest approach to the center of the galaxy. If $\alpha_{\text{nr}} \ll 1$ is the angle through which the trajectory is deflected, show that

$$\alpha_{\text{nr}}(v) = \frac{2Gb}{v^2} \int_b^{\infty} \frac{dr}{\sqrt{r^2 - b^2}} \frac{M(r)}{r^2}. \quad (1)$$

- b. (5 points) This result holds only for non-relativistic particles, but in general relativity it can be shown that the deflection angle for photons is

$$\alpha = 2\alpha_{\text{nr}}(v = c). \quad (2)$$

Assume that the galaxy has a flat rotation curve with rotation speed v_c , i.e. that its rotation speed is independent of r . Show that the deflection angle for photons is

$$\alpha = \frac{2\pi v_c^2}{c^2}. \quad (3)$$

- c. (5 points) If space is Euclidean, show that the angular separation of two images formed by a galaxy with a flat rotation curve is

$$\Delta\theta = \frac{2\alpha D_{\text{sl}}}{D_s} \quad (4)$$

where D_{sl} is the distance between the source and the lens (i.e. the galaxy) and D_s is the distance between the source and the observer.

- d. (2 points) For many (elliptical) galaxies, a good approximation to the density distribution is that of a so-called singular isothermal sphere:

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}, \quad (5)$$

where σ is the velocity dispersion. Show that for such a density profile the angular image separation becomes

$$\Delta\theta = \frac{8\pi D_{\text{sl}} \sigma^2}{D_s c^2}. \quad (6)$$