

STATE UNIVERSITY OF NEW YORK AT STONY BROOK  
DEPARTMENT OF PHYSICS AND ASTRONOMY

Part I.

Tuesday, 31 August 1999 – Day 1

Comprehensive Examination in Classical Mechanics and Special Relativity  
and in Electromagnetism and Optics

**General instructions:** In each of the two areas, do two of the three problems. Each problem should take about  $\frac{3}{4}$  hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

Classical Mechanics and Special Relativity

Three problems, work any two.

CM I. (Sorge) \*\*\*\*\*PUT IN FIGURE\*\*\*\*\*

In the presence of the earth's gravity (magnitude of acceleration  $g$ ), a pointlike object of mass  $2m$ , which is connected via a frictionless pulley to another mass  $m$ , drops down from a height  $h$ . Then it enters a liquid that creates a frictional force  $F_{fr} = -kv$  that proportional to the object's velocity (magnitude  $v$ ) with constant of proportionality  $k$ .

- a. Write down the Lagrange function for the position change  $y$  of any of the masses before the object is submerged in the liquid. What is the equation of motion after the mass has entered the liquid?
- b. Discuss why on physical grounds the velocity of the object approaches a limit  $v \rightarrow v_\infty$  and, therefore, the long-time behavior of the solution for  $y$  is expected to be

$$y(t) \approx v_\infty \cdot t \quad . \quad (1)$$

Show that Eq. (1) is, indeed, a particular solution of the equation of motion. Determine the value of  $v_\infty$ .

- c. Determine the general solution of the equation of motion. Specify its yet unknown constants such that the initial conditions of the problem are fulfilled:

$$\begin{aligned} y(t=0) &= 0 \\ \dot{y} = v(t=0) &= 0 \quad . \end{aligned}$$

CM II. (Fossan)

In the presence of the earth's gravity (magnitude of acceleration  $g$ ), consider a particle of mass  $\mathbf{m}$  that rotates with a constant angular velocity  $\omega$  about a vertical axis  $z$ . Space coordinates in the horizontal plane  $x$  and  $y$  are related by  $y = x \tan(\omega t)$ . This constraint in cylindrical coordinates can be rewritten as  $\phi = \omega t$ .

- a. Find the equations of motion of the particle (Lagrange multipliers can be useful).
- b. Solve the equations of motion by transforming to cylindrical coordinates.

CM III. (Weisberger)

In an inertial frame a plane electromagnetic wave of wavelength  $\lambda = 700$  nm propagates in the positive  $x$ -direction. What is the wavelength measured by an observer moving with a velocity

- a.  $\vec{v} = (c/2)\hat{e}_x$  in the direction of the wave?
- b.  $\vec{v} = -(c/2)\hat{e}_x$  in the direction opposite to the wave?
- c.  $\vec{v} = (c/2)\hat{e}_y$  transverse to the wave?

What is the direction of propagation of the wave as seen by the observer in part c.?

## Electricity and Magnetism and Optics

Three problems, work any two.

EM&O I. (Engelmann) \*\*\*\*\*PUT IN FIGURE\*\*\*\*\*

A long, fixed solenoid with  $N$  turns per unit length, a current  $I$ , and radius  $R$  is concentric with two non-conducting cylindrical shells of radii  $a$  and  $b$ , respectively, and length  $l \gg b > R > a$ . The cylinders have negligible thickness, are free to rotate without friction about the  $z$ -axis, and have fixed charges ( $+Q$  on the outer cylinder and  $-Q$  on the inner cylinder) uniformly distributed over their outer surfaces. The current  $I$  is switched off and the cylinders start to turn.

- a.** Find the angular momentum (magnitude and direction) of each cylinder after the current is switched off. Express your answer in terms of  $Q$ ,  $N$ ,  $I$ ,  $R$ , and  $a$ .
- b.** Make a statement concerning conservation of angular momentum in this problem. Do a calculation to justify your answer.

In high-energy physics, MWPCs (multi-wire proportional chambers) have become standard tools to measure trajectories of charged particles. Conducting anode wires are located at a distance  $a$  from a conducting cathode plane (see figure). A charged particle traversing the MWPC ionizes the detector gas. The electrons are collected and amplified at the nearest anode wire, resulting in a point-like charge  $q$  that hits that anode wire and is detected. The wire number gives the  $y$ -coordinate of the traversing particle. In order to measure the  $x$ -coordinate, the cathode plane (assume at ground potential) can be segmented into strips of width  $d$ , perpendicular to the anode wires. Assume that the width of the cuts between the strips is negligible.

- a. Just before the charge  $q$  hits an anode wire, what is the charge density distribution induced on the cathode plane, as a function of the anode-cathode distance  $a$ ? In this calculation neglect the effect of the anode wires.
- b. What is the total charge induced on the cathode plane? Prove your answer.
- c. The position resolution in the  $x$ -direction depends on the width of the cathode strips. Given the fact that optimum resolution is achieved if  $\sim 50\%$  of the charge on the cathode plane is induced on a disk whose radius is equal to the strip width  $d$ , derive an expression for the optimum strip width  $d_{opt}$  as a function of  $a$ .

EM&O III. (de Zafra)

An oil film of thickness  $d$  and refractive index  $n_f = 1.40$  coats a flat glass plate that has a refractive index  $n_g = 1.50$ . White light is incident (through air) on the oil-glass surface at normal incidence.

- a.** What is the minimum value of  $d$  if green light (vacuum wavelength  $\lambda_0 = 500$  nm) is found to be reflected back strongly?
- b.** Re-evaluate this problem if the oil is not coating the glass plate but is
  - i.* floating on water ( $n_w = 1.33$ )
  - ii.* floating on liquid mercury (the element Hg).
- c.** In part **a.**, assume  $d = 400$  nm. At what angle of incidence  $\theta_I$  will yellow light ( $\lambda_0 = 600$  nm) be most strongly reflected?