STATE UNIVERSITY OF NEW YORK AT STONY BROOK
DEPARTMENT OF PHYSICS AND ASTRONOMY

Part I.

Tuesday, 18 January 2000 – Day 1

Comprehensive Examination in Classical Mechanics and Special Relativity and in Electromagnetism and Optics

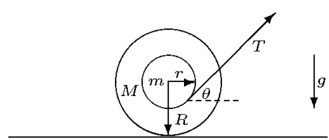
General instructions: In each of the two areas, do two of the three problems. Each problem should take about $\frac{3}{4}$ hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

Classical Mechanics and Special Relativity

Three problems, work any two.

CM I. (Behrend)

A spool is made of two uniform disks, each of mass M and radius R, joined rigidly by a uniform hollow cylinder of mass m and radius $r < R$. The axis of the cylinder passes through the center of and perpendicular to the plane of each disk. Thread having negligible mass and thickness is wound around the central cylinder. The spool rests on a horizontal table and the free end of the thread is pulled by a force T acting perpendicular to the axis of the spool and at a fixed angle θ above the horizontal; see the figure.



a. Find the condition on T for the spool not to move vertically.

b. Find the moment of inertia of the spool about its axis.

c. Assuming that the spool rolls along the table without slipping, show that thread may either wind onto, or unwind from, the spool, and find the conditions on θ for each to occur.

d. Find the condition on the coefficient of static friction between the table and spool for the rolling in fact to occur without slipping.

CM II. (Belitsky)

Consider a linear chain of three atoms (a molecule) with masses m_1, m_2, and m_3 on each site and separated by the distances ℓ_1 and ℓ_2 as shown in figure. The elasticities of the connecting springs are k_1 and k_2, respectively, in the longitudinal direction and k_3 in the transverse (bending) direction.

a. (6 pts.) Count the number of independent longitudinal and transverse oscillations of this molecule. (Do not include rotations).

b. (7 pts.) Consider the case $m_1 = m_3 \equiv m \neq m_2$; $\ell_1 = \ell_2 \equiv \ell$; and $k_1 = k_2 \neq k_3$.

1. Find the Lagrangian describing the oscillations.

2. Find the frequencies of the longitudinal oscillations.

3. Find the frequencies of the transverse oscillations.

Hint: for transverse oscillations the potential energy $U = \frac{k_3 \ell^2}{2} \delta^2$ is given by the deviation δ expressed in terms of transverse displacements of atoms via $\delta = \delta_1 + \delta_2 = \frac{1}{\ell}\left[(y_1 - y_2) + (y_3 - y_2)\right]$, where $\ell = \frac{1}{2}(\ell_1 + \ell_2)$. See the figure.

c. (7 pts.) Now consider the different case $m_1 \neq m_2 \neq m_3$; $\ell_1 \neq \ell_2$; and $k_1 \neq k_2 \neq k_3$.

1. Find the potential energy of the molecule.

2. Find the frequencies of the longitudinal oscillations.

3. Find the frequencies of the transverse oscillations.

NOTE: The parameters we have introduced differ in different parts of the problems. Oscillations are assumed to be small, $|\Delta x| \ll x$.

CM III. (Vogelsang)

A high-energy particle collision produces a π^0 of kinetic energy T (as measured in the laboratory frame).

a. Determine the magnitude of the pion's momentum $|\vec{p}|$, and its speed $|\vec{v}|$, in terms of T and the π^0 mass m.
Give a number for $\beta \equiv |\vec{v}|/c$, if $T = 2$ GeV. The π^0 mass is $m = 135$ MeV$/c^2$; c is the speed of light.

b. The π^0 decays into two photons ($\pi^0 \to \gamma\gamma$) with a proper lifetime of $\tau = 8.4 \times 10^{-17}$ s. For the present example, what is the lifetime as seen from the laboratory frame? What distance does the π^0 travel in this time?

c. What is the smallest possible angle (in the laboratory frame) between the two decay photons? What are the energies of the two photons when produced at this angle?

Electricity and Magnetism and Optics

Three problems, work any two.

EM&O I. (Kahn)

A sphere of radius R contains charge of uniform density ρ for $r < R$.

a. (10 pts.) Find the potential $\varphi(r)$ for all r. Express your answers in terms of ρ, ϵ_0, and R.

b. (5 pts.) Plot $\varphi(r)$ as a function of r for $0 < r < 3R$.

c. (5 pts.) Plot the magnitude of the electric field $E(r)$ as a function of r for $0 < r < 3R$.

In the plots of parts **b.** and **c.**, choose a scaling in which the quantity (ρ/ϵ_0) is numerically equal to 1 and $R = 2$ (in arbitrary units). Show explicity how the potential and the field behave at $r = R$.

EM&O II. (Goldhaber)

An electron is projected straight upward in the Earth's (constant) gravitational field. At $t = 0$ the electron reaches the top of its trajectory and then falls freely for 2 minutes. (In the following neglect air friction.)

a. (7 pts.) Write down an expression (correct to factors of order unity) for the power of the electromagnetic radiation from the falling electron.

b. (7 pts.) Using the result of part **a.**, find the fraction of the work done by gravity in the 2-minute interval that is converted into electromagnetic radiation. Evaluate this fraction numerically.

c. (6 pts.) Comment on the significance of the radiation loss calculated in part **b.** and give an example of circumstances in which the loss would be more significant.

EM&O III. (Metcalf)

A radiotelescope, made up of a parabolic dish of diameter d and a small detecting antenna near the focal point, is located on a mountaintop, a distance $h = 1000$ m above sea level. It is tuned to receive 1.5 GHz signals from a distant point radio source that is located an angular distance θ above the horizon. Waves from the source reach the antenna by two paths: path 1 is direct, and path 2 is via a 100% reflection from the surface of the water, as shown in the figure. The waves from these two paths interfere in the (horizontally polarized) detecting antenna.

- **a.** (7 pts.) If the intensity of the waves along each path is I_0, derive an expression for the total received intensity as a function of θ. (Neglect effects of the earth's curvature, atmosphere, etc.)
- **b.** (3 pts.) Explain (in words) how your answer for part **a.** satisfies energy conservation.
- **c.** (7 pts.) Suppose two point radio sources of equal intensity are separated by a small angle $\Delta\theta$. Determine the smallest value of $\Delta\theta$ that can be resolved by recording the total intensity variation of the sources as they rise above the horizon.
- **d.** (3 pts.) Compare your answer in part **c.** with the intrinsic (single path) resolution of the telescope if the dish diameter is $d = 10$ m.