STATE UNIVERSITY OF NEW YORK AT STONY BROOK
DEPARTMENT OF PHYSICS AND ASTRONOMY

Part I.

Wednesday, 19 January 2000 – Day 2

Comprehensive Examination in Quantum Mechanics and in Statistical Mechanics and Thermodynamics

General instructions: In each of the two areas, do two of the three problems. Each problem should take about $\frac{3}{4}$ hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

Quantum Mechanics

Three problems, work any two.

QM I. (Likharev)

At $t = 0$ a 1D harmonic oscillator of a mass m and frequency ω was in the state

$$\mid \alpha \rangle = \frac{1}{\sqrt{2}} \left(\mid 3 \rangle + \mid 4 \rangle \right),$$

where $\mid n \rangle$ are stationary states corresponding to eigenenergies $E_n = \hbar\omega(n + \frac{1}{2})$.

a. Find the average energy $\langle E \rangle$ of the oscillator and its r.m.s. uncertainty.

b. Find the initial expectation values of the oscillator coordinate and momentum.

c. Find the time evolution of $\langle x \rangle$, $\langle p \rangle$, and $\langle E \rangle$ at $t > 0$.

Hint: for the oscillator, the coordinate matrix elements are

$$\langle n' \mid x \mid n \rangle = \sqrt{\frac{\hbar}{2m\omega}} (\sqrt{n}\, \delta_{n',n-1} + \sqrt{n+1}\, \delta_{n',n+1});$$

this equation may also be used to derive the matrix elements of momentum.

QM II. (Stephens)

A particle of mass m is confined inside of a sphere of radius R.

a. (5 pts.) Estimate the energy E_0 of its ground state from the uncertainty relation.

b. (15 pts.) Find E_0 by solving the Schrödinger equation.

QM III. (Verbaarschot)

The time reversal operator T satisfies the relation

$$T\sigma_k T^{-1} = -\sigma_k,$$

where σ_k, $k = 1, 2, 3$, are the Pauli spin matrices.

a. Give a one-line, intuitive interpretation of this relation.

b. Show that $T = i\sigma_2 K$, with K the complex conjugation operator, satisfies this relation.

c. Show that, if the Hamiltonian H commutes with T, i.e., $[H, i\sigma_2 K] = 0$, all eigenstates of the Hamiltonian are at least doubly degenerate. To prove this consider the action of $i\sigma_2 K$ onto an eigenstate of H. For Hamiltonians that are not invariant under rotations this is the so-called Kramers degeneracy.

Statistical Mechanics and Thermodynamics

Three problems, work any two.

SM&T I. (McCoy)

Consider a one-dimensional, classical gas of N hard rods, each of length a, with the pair-potential

$$V(r) = \begin{cases} \infty & \text{if } -a \leq r \leq a \\ 0 & \text{otherwise.} \end{cases}$$

The system is placed on a ring of circumference L.

a. (8 pts.) Show that the second virial coefficient is a.

b. (8 pts.) Compute the exact equation of state.

c. (4 pts.) Compare the results of parts **a.** and **b.** by expanding the equation of state in the density.

SM&T II. (Aleiner)

Consider the two-dimensional (2d) Fermi-gas made of N non-relativistic particles at temperature $T = 0$, confined in a box with (2d) volume V_0. Then, one wall of the box is removed and the system is allowed to expand into very large box, with final (2d) volume $V \gg V_0$.

a. (9 pts.) Find the final temperature of the gas.

b. (6 pts.) How much entropy was produced?

c. (5 pts.) If the particles are adiabatically forced back into the smaller box, what is its temperature now? (Indicate a method of calculation, but you are not supposed to calculate it explicitly.)

SM&T III. (Zahed)

Two identical bodies with a heat capacity C that is independent of temperature T are used for heating–cooling in a non-cyclic heat engine. The heating–cooling process is carried out at constant pressure, and there is no phase change. The initial temperatures of the two bodies are T_1 and T_2, respectively, while the interaction with the heat engine brings them both to the same temperature T_f at the end.

a. (6 pts.) Evaluate the total heat transferred to the engine.

b. (8 pts.) How does the second law of thermodynamics restrict possible values of T_f, in terms of T_1 and T_2?

c. (6 pts.) What is the maximal amount of work that can be extracted from such an engine, with given T_1 and T_2?