STATE UNIVERSITY OF NEW YORK AT STONY BROOK
DEPARTMENT OF PHYSICS AND ASTRONOMY

**Part II.**

**Thursday, 20 January 2000 – Day 3**

**Comprehensive Examination in "Experiment" and "Breadth"**

**General instructions:** Twelve problems are given. You should do any four, subject to the constraint that you should answer **no more than three** from "experiment" and **no more than three** from "breadth" (that is, not all four problems can be chosen from the same category). Each problem should take about $\frac{3}{4}$ hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

## "Experiment"

Experiment I. (Orozco)

The Nobel Prize for Chemistry in 1999 went to Ahmed H. Zewail from Cal Tech for the development of femtosecond spectroscopy (FS) to study transition states of chemical reactions.
FS is similar to the "strobe technique" used in high-speed photography to "freeze" the movement of an object. By taking successive "pictures" one makes a "movie".

**a.** Estimate how long it takes for a molecular system to dissociate if you know that a typical separation for a molecular bond is $10^{-10}$ m and the reaction products fly away with speeds of about $10^3$ m/s.

**b.** What would be the "exposure time" needed to freeze the motion of the molecules as they fly away such that the resolution of the "picture" is better than 1/100 of the typical bond separation?

**c.** Just as in the strobe technique, there is a trigger to set the objects in motion and later a flash of light to see what happened. In FS the trigger is a laser pulse, and the flash of light is a second short pulse. What is the bandwidth of electromagnetic radiation in the laser pulse needed to attain the resolution specified in part **b.**?

**d.** Normally FS uses only one laser to produce a short pulse that is then split into two. What kind of length delay and length control is necessary to be able to map what happens during a chemical reaction if you want to "record" it for the time it takes for the system to move a distance of five molecular bonds with a resolution of 1/100 of the bond length.

**e.** There are no direct electronics means to measure the length of FS laser pulses. What optical technique could you use to be able to do it with a resolution of 1/10 of the width of the pulse? Make a sketch of the apparatus and briefly describe how it would work.

Experiment II. (Goldman)

A helium dilution refrigerator works by circulating $^3$He through a condensed mixture of the isotopes $^3$He and $^4$He. At temperatures below 0.8 K the mixture of the two isotopes separates into two distinct liquid phases: one floats on top of the other in a field of gravity, such as is the case on Earth. At very low temperatures one phase ("concentrated" phase) is nearly 100% pure $^3$He; the other phase ("dilute phase) has about 6.4% $^3$He, with the rest $^4$He. During operation of the refrigerator, $^3$He is forced (by an external pump) to cross the concentrated–dilute phase boundary, a process somewhat analogous to evaporation, which causes cooling due to absorption of latent heat. The place where the phase boundary is maintained is called the "mixing chamber".

**a.** (4 pts.) Which phase is the upper layer, and why?

Imagine you have the mixing chamber maintained at a temperature $T_m = 10$ mK. Now, you want to vary the temperature of an "experiment" $T_e$ up to 200 mK. The experiment is located in a chamber fully isolated from the outside world, except that it is connected to the mixing chamber by an $L = 30$ cm copper wire that is $d = 0.1$ mm in diameter. In order to maintain $T_e > T_m$, there is a resistive heater wound of the same copper wire with length $L_h = 120$ cm.

**b.** (6 pts.) Derive the dependence of $T_e$ on the heater current $I$. Use plausible simplifying assumptions and assume $T_m$ is constant. (Use the Wiedemann-Franz law that relates the electrical and thermal conductivities to each other).

**c.** (4 pts.) Sketch $T_e$ versus $I$.

**d.** (6 pts.) If the maximum cooling power provided by the mixing chamber is $P = 0.1\ \mu$W while it still maintains $T_m$=10 mK, find the heater current required to heat the experiment to 200 mK. Use the experimentally determined Lorentz number $\Lambda = 2.5 \times 10^{-8}$ WOhm/K$^2$.

Experiment III. (Engelmann)

This problem concerns measurements made on a radioactive sample that emits $\gamma$ rays with three different energies $E_1 < E_2 < E_3$.

**a.** (4 pts.) You measure the photon energies with NaI crystal detector(s). For one photon energy sketch a plot (label the axes!) of the detected energy spectrum, explain which two dominant processes are responsible for the photon detection, and indicate where they show up on your plot.

**b.** (2 pts.) You notice that one photon energy is (at least close to) the sum of the energy of the two other photons. You want to confirm this with better energy resolution using Ge semiconductor detector(s). What is the order of magnitude improvement of the energy resolution of a Ge detector over the NaI detector? Give reason(s) for this improvement.

**c.** (3 pts.) You confirmed the "sum rule" from **b.** via your measurements with the Ge detector(s) and now suspect that the three $\gamma$ rays are the result of transitions between three nuclear states where the higher energy states feed the lower states. Make a sketch of a scheme of nuclear energy levels that could do this. Label the energy levels with $e_1$, $e_2$, and $e_3$, respectively. On your energy level scheme indicate the transitions giving rise to the photons and label them with the photon energies $E_1$, $E_2$, and $E_3$, respectively.

**d.** (5 pts.) You have equipment for the measurement of photon–photon coincidences. Assuming that the lifetime of the intermediate nuclear state is small compared to the resolving time of your equipment, you look for coincidences but find none. Describe (briefly!!) the basic functionality of a coincidence circuit. Make sure you also define "resolving time".

**e.** (3 pts.) You now suspect that the intermediate ("fed") state (and maybe the "feeding" state) have lifetimes large compared to the coincidence resolving time of your setup. For the two lower energy photons you modify the setup so you can measure the arrival time of one photon relative to the arrival time of the other photon. Your setup covers time intervals large compared to the resolving time in **d.**, and you find many such events. Is your nuclear energy level scheme now uniquely determined? Explain.

**f.** (3 pts.) You want to find out whether the feeding state lifetime is also large. You get freshly prepared sample(s) (irradiated, say, by your in-house accelerator). You measure the ratio "fed activity" over "feeding activity" and find this ratio to grow with with increasing time after the irradiation that prepared your sample(s). Which is the larger, the lifetime of the "fed state" or the lifetime of the "feeding state".

Experiment IV. (Yanigasawa)

In a large underground water Cherenkov detector such as Super-Kamiokande, charged particles above the Cherenkov threshold emit photons into a cone with the Chrenkov angle $\theta_c$ with respect to the particle direction. The angle is defined by $\cos\theta_c = 1/(n\beta)$ where $n$ is the index of refraction of the medium and $\beta$ is the velocity of the particle in units of c, the speed of light in vacuum. Photomultipliers (PMTs) covering the walls of the water tank detect the Cherenkov photons in a ring like pattern. For a muon or pion (muon-like) the pattern is a relatively sharp ring. An electron or $\gamma$ (electron-like) creates an electromagnetic shower immediately after its production thus leading to a more blurred ring pattern. Hence one can distinguish between electron-like and muon-like particles. The shower is contained in a region that is small relative to the size of the tank. The measurement of time of flight of the Cherenkov photons allows the determination of the particle origin (vertex). The measurement of the total PMT charge due to the Chrenkov photons yields the particle momentum. Your goal is to use this detector to search for proton decay in the mode $p \to e^+ + \pi^0$.

**a.** (3 pts.) Use the above paragraph to describe the event signature.

**b.** (3 pts.) For a proton lifetime of $10^{33}$ years, calculate the amount of water needed to detect 10 events in one year of operation.

**c.** (4 pts.) Calculate the momenta of the $e^+$ and $\pi^0$ in terms of the proton and $\pi^0$ mass. (Ignore the positron mass and Fermi motion.)

**d.** (4 pts.) You want to use relativistic muons for calibration purposes. Calculate the Cherenkov threshold ($\theta_c = 0$) for muons in terms of the muon mass and the index of refraction.

**e.** (6 pts.) The dominant background to the proton decay above is neutrino induced, $\nu_e + N \to e^- + N' + \pi^0 + k\pi^\pm$ ($k$ is an integer $\geq 0$), where the momenta of $\pi^\pm$s are below the Cherenkov threshold and $N$ and $N'$ are nuclei that are invisible (i.e., undetected). Describe how you attack this background using event signature and/or kinematics.

Experiment V. (Lanzetta)

In an expanding universe, bolometric surface brightness $S$ (i.e. energy flux per unit solid angle) depends on redshift as $S \propto (1+z)^{-4}$, and specific surface brightness per unit wavelength interval $S_\lambda$ (i.e. specific energy flux density per unit wavelength interval per unit solid angle) depends on redshift as $S_\lambda \propto (1+z)^{-5}$. These results are independent of cosmological model.

**a.** Consider a face-on spiral galaxy of redshift $z = 1.0$ and characteristic radius $R = 11\,\mathrm{kpc}$. For some choice of cosmological model, this characteristic radius corresponds to an angular radius of $\theta = 0.5$ arcsec. The specific energy flux density of the galaxy at observed-frame wavelength $\lambda = 7000$ Å within the characteristic radius is measured to be $F_\lambda = 3 \times 10^{-18}$ erg s$^{-1}$ cm$^{-2}$ Å$^{-1}$. What is the mean specific surface brightness of the galaxy within the characteristic radius at observed-frame wavelength $\lambda = 7000$ Å?

**b.** The energy flux of the galaxy in redshifted [O II] $\lambda 3727$ emission is measured to be $F = 10^{-16}$ erg s$^{-1}$ cm$^{-2}$. What is the mean surface brightness of the galaxy within the characteristic radius in redshifted [O II] $\lambda 3727$ emission?

**c.** Now consider the same galaxy at redshift $z = 6$. Assume that the [O II] $\lambda 3727$ emission to Ly$\alpha$ line emission energy flux ratio of the galaxy is characterized by $F([\mathrm{O\ II}])/F(\mathrm{Ly}\alpha) = 0.2$. What is the mean surface brightness of the galaxy within the characteristic radius in redshifted Ly$\alpha$ emission?

**d.** The most sensitive spectroscopic observations yet obtained by the Space Telescope Imaging Spectrograph on the Hubble Space Telescope are characterized by a $1\sigma$ single pixel energy flux sensitivity of around $1 \times 10^{-18}$ erg s$^{-1}$ cm$^{-2}$. For these observations, the angular pixel scale is 0.05 arcsec pixel$^{-1}$, and the spectral pixel scale is 5 Å pixel$^{-1}$. Assume that the same galaxy at redshift $z = 6$ is observed through an aperture of 0.5 arcsec length and 0.2 arcsec width (assume a uniform surface brightness over this aperture equal to its mean surface brightness) and that the Ly$\alpha$ emission line of the galaxy is spectrally unresolved. Is the galaxy detected in Ly$\alpha$ line emission?

**e.** Now that you have used the results stated at the start, derive that $S$ depends on redshift as $S \propto (1+z)^{-4}$ and that $S_\lambda$ depends on redshift as $S_\lambda \propto (1+z)^{-5}$ (for wavelength-independent specific luminosity density, i.e., $L_\lambda \propto \lambda^0$).

Experiment VI. (Simon)

*Possibly useful parameter:* 1 parsec = $3.1 \times 10^{16}$ m.

**a.** (7 pts.) The absorption of starlight by dust in the Milky Way is very non-uniform, but along clear lines of sight in the V-band it has the approximate value 1 mag per kpc. In other words, light at 5500Å is attenuated by about $e^{-1}$ in 1000 pc. The wavelength dependence of the attenuation indicates that the radius of the dust particles is about 0.2 $\mu$m. The distribution of gas in the Milky Way is also non-uniform, averaging about 1 H atom/cm$^3$.

If the Earth's atmosphere had the same kind of dust particles and the same relative abundance of gas and dust by number, how far would we able to see?

**b.** (7 pts.) $\alpha$ Nuti is the brightest and only star in the obscure constellation the "Test-tube Holder". Its epoch 2000 coordinates are, fortuitously, RA(2000)=04h 00m 00.00s and Dec(2000)= +75° 00' 00.0". Its brightness at V is 4.1 mag and its color is $(B-V) = 0.00$. The satellite *HIPPARCOS* measured its parallax to be 0.015".

Give the spectral type, including luminosity class, of $\alpha$ Nuti. When, if ever, is $\alpha$ Nuti observable from Stony Brook? Why is it necessary to specify the epoch at which its coordinates are given?

**c.** (6 pts.) The Sun is composed mostly of H and He, yet by far the strongest spectral lines in its visible light spectrum, the so-called Fraunhofer spectrum, are the lines of ionized Ca. The hydrogen Balmer lines are present but at a strength comparable to the lines of, say, neutral Mg.

In a few sentences, and perhapswith a simple quantitative estimate, explain why the Balmer lines are relatively weak and how astronomers came to understand that the Sun is mostly H and He. After these two elements, what are four next most abundant elements in the Sun, by number?

## “Breadth”

Breadth I. (Koch)

Consider atomic states (and transitions between them) described by so-called $LS$ coupling. Taking the index $i$ for each of the atomic electrons, the orbital angular momenta $\ell_i$ couple together to make $L$, the spin angular momenta $s_i$ couple together to make $S$, and $L$ and $S$ couple together to make $J$. The $LS$-coupled state is denoted by (filled orbitals) $^{2S+1}L_J$, where each filled orbital $n\ell$ is labeled by its principal quantum number $n$, $\ell$, and superscript giving the number of electrons in that orbital. For example, the ground state of Li is $(1s^2 2s)\ ^2S_{1/2}$. Now consider the argon atom $A$, for which the nuclear charge $Z = 18$.

**a.** (2 pts.) For its ground state, give the full notation (filled orbitals) $^{2S+1}L_J$.

**b.** (3 pts.) Now make a “K-shell vacancy” in the $A$ atom by promoting one of its innermost (K-shell: $n = 1$) electrons to the continuum, i.e., ionizing it. Before the $A^+$ system (consider only the bound electrons) has a chance to “relax” (i.e., it still has a vacancy in its K-shell), write down for it the full notation (filled orbitals) $^{2S+1}L_J$.

**c.** (3 pts.) Why is the system in **b.**, which has a vacancy in its K-shell, highly-excited? Why/how can it decay by emitting x-ray(s)?

**d.** (6 pts.) Estimate to a factor of 2 or better in eV the x-ray energy emitted by the system in **b.**, which has a vacancy in its K-shell. Does it make any important difference to your estimate if the electron that was promoted out of the K-shell to make the K-vacancy went all the way to the continuum (making an $A^+$ ion) or remained bound in an outer level (keeping the system an A atom)? State whether your estimate for the x-ray energy is likely to be too big or too small and give reason(s) why.

**e.** (6 pts.) Is it possible for an $A$ atom having a vacancy in its K-shell to decay without emitting photon(s)? Give an example of such a decay, being careful to give the full notation (filled orbitals) $^{2S+1}L_J$ for both the initial and final states of the system with all 18 electrons.

Useful number: binding energy of the ground state of the hydrogen atom is 13.6 eV.

Breadth II. (Allen)

The alloy $Cu_{0.5}Zn_{0.5}$ ($\beta$-brass) crystallizes at high temperature in the bcc (body-centered cubic) structure, with Cu and Zn atoms randomly distributed. The side of the conventional cube is $a = 0.30$ nm.

**a.** The crystal is oriented with the (001) axis vertical, and x-rays of wavelength 0.100 nm are incident. What is the smallest value of $\sin\theta$ at which specular reflection occurs at high temperature? ($\theta$ is conventionally measured not from the normal, but from the surface.)

**b.** When the temperature $T$ is lowered below 740 K, specular reflection also begins to be seen at values of $\sin\theta$ smaller by 2 than were seen at higher temperatures. What has happened?

**c.** The conductivity $\sigma$ is proportional to the squared plasma frequency $\omega_P^2 = 4\pi e^2 (n/m)_{\text{eff}}$. At $T < 740$ K the plasma frequency diminishes. Why? Nevertheless, the conductivity increases. Why?

Breadth III. (Shuryak) FIGURE!!!

**a.** (6 pts.) $^{158}$Dy is a deformed nucleus. Assuming it is an axially symmetric ellipsoid, with one axis longer than the other two (take $R_{long}/R_{short} = 1.3$), made of homogeneous nuclear matter with nucleon density $n_0 = 0.16$ fm$^{-3}$, show that its moment of inertia is $I = M(R^2_{long} + R^2_{short})/5$ if it rotates as a solid body.

**b.** (6 pts.) Measured energies of the excited states versus angular momentum $J$ are shown in the figure below. Evaluate the moment of inertia both above and below the break in the data. Compare both results to part **a.**

**c.** (8 pts.) Going into a frame rotating with an angular velocity $\omega$, one should modify the Hamiltonian as follows: $H \to H - \vec{\omega}\vec{j}$, where $j$ is the angular momenta of individual nucleons. So, $\omega$ acts like a magnetic field. Evaluate the numerical value of this correction term, for a typical $j = 7/2$, at the breaking region $J = 14$. Compare it to the $pp$, $nn$ pairing energy $\epsilon_{pair} \sim 1$ MeV. What does it tell you about the reason for the change in moment of inertia at the break? Why was it small at low $J8$ (i.e., below the break)?

*Excitation energy (MeV) versus $J(J+1)$ where $J$ is the orbital angular momentum.*

Breadth IV. (Jung)

**a.** (3 pts.) Calculate the decay distance of a 10 GeV muon. The lifetime of muon is 2.2 $\mu$s and the mass of the muon is 106 MeV/c$^2$.

**b.** (5 pts.) Charged pions ($\pi^+$) decay to $\mu^+\nu_\mu$ at 99.99%. Draw a quark level Feynmann diagram of this decay. Explain qualitatively why their decay to $e^+\nu_e$ is so much suppressed despite the fact that the electron is much lighter than the muon.

**c.** (8 pts.) In June 1998, the Super-Kamiokande collaboration announced observation of neutrino oscillation phenomena. The observation was made by analyzing events induced by atmospheric neutrinos. The atmospheric neutrinos are produced in the atmosphere about 15-20 km above ground, and the energy spectrum of the neutrino flux peaks at about 1 GeV. The predicted flavor flux ratio $\nu_\mu/\nu_e$ is about two for low energy neutrinos ($E_\nu < 5$ GeV) without neutrino oscillations. The experiment, however, observed the ratio to be about one which indicates that $\nu_\mu$'s are somehow disappearing, possibly due to neutrino oscillations. Explain why this ratio is predicted to be about two. What is your prediction of the ratio (greater than two or less than two) for the high energy neutrinos ($E_\nu > 10$ GeV) and why?

**d.** (4 pts.) Postulating that the lack of measured $\nu_\mu$ flux is due to $\nu_\mu$ to $\nu_\tau$ oscillation, how do you propose to further verify this hypothesis?

(Hint: The oscillation probability P($\nu_\mu \rightarrow \nu_\tau$) is given to be $\sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{E}$, where $\theta$ is a mixing angle between the mass eigenstates and the weak eigenstates of the neutrinos, $\Delta m^2$ is the mass squared difference between the two mass eigenstate masses, $L$ is the neutrino flight distance, and $E$ is the neutrino energy.

Breadth V. (Yahil)

This question considers peculiar velocities at the present epoch and their use in determining the cosmological density parameter, $\Omega$. (The cosmological constant, $\Lambda$, is not important in this context and is ignored.)

**a.** Show that the product of the Hubble constant and the age of the universe, $H(t)t$, takes the value of $2/3$ for an Einstein-de Sitter universe ($\Omega = 1$) and 1 for an empty universe($\Omega = 0$).

**b.** Consider a spherical void in which the density is zero. This "bubble" expands faster than the rest of the universe. Assuming that the background universe is an Einstein-de Sitter universe, show that the relative peculiar velocity in the bubble is

$$v/Hr = 1/2 \quad . \tag{1}$$

**c.** Show that linear perturbation theory, applied to this not so small perturbation, predicts the relative peculiar velocity to be

$$v/Hr = 1/3 \quad . \tag{2}$$

The perturbation equation in linear theory is $\vec{\nabla} \cdot \vec{v} = -Hf(\Omega)\delta$, where $\vec{v}$ is the perturbation in velocity, $v = |\vec{v}|$, and $\delta \equiv \Delta\rho/\rho$ the density perturbation; $f(\Omega)$ increases monotonically with $\Omega$, and $f(1) = 1$.

**d.** Explain why the measurement of the peculiar velocities of galaxies around an observed void (no galaxies) provides only a lower limit to the cosmological density parameter, $\Omega$.

**e.** What are some of the observational difficulties in using this technique to determine a lower limit on $\Omega$?

Breadth VI. (Solomon)

It is generally assumed by Radio Astronomers that observations of the 21 cm line in emission will yield the total column of interstellar neutral atomic hydrogen along a line of sight through the Milky Way. The column density of hydrogen [$cm^{-2}$] depends only on the intensity of the line integrated over frequency and is independent of the gas kinetic temperature and the local density [$cm^{-3}$]. (Units can be in meters or cm.)

- **a.** (3 pts.) Under what conditions is the above true?
- **b.** (3 pts.) What transition does the 21 cm line correspond to? What are the statistical weights of the upper and lower level?
- **c.** (9 pts.) Derive the expression for the integrated intensity of the 21 cm line and show that it is independent of temperature under the above conditions. Express the intensity in terms of brightness temperature, and express the integrated intensity in terms of velocity integrated brightness temperature [K km/s]. Assume that the total absorption crossection (per atom) integrated over frequency $\sigma_{\textbf{total}}$ is known.
- **d.** (2 pts.) Why is the integrated intensity also independent of local density, in the range of densities normally encountered in atomic H clouds?
- **e.** (3 pts.) Under what conditions does the temperature independence break down? (Under what condtions does the intensity depend on the gas temperature?)