STATE UNIVERSITY OF NEW YORK AT STONY BROOK
DEPARTMENT OF PHYSICS AND ASTRONOMY
Part I.
Tuesday, 5 September 2000 – Day 1
Comprehensive Examination in Classical Mechanics and Special Relativity and in Electromagnetism and Optics

General instructions: In each of the two areas, do two of the three problems. Each problem should take about $\frac{3}{4}$ hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

Classical Mechanics and Special Relativity

Three problems, work any two.

CM I. (Mihaly)

Three small massive balls (m_1, m_2, m_3) are connected by massless rods of lengths l_1, l_2, and l_3. The balls are supported by a horizontal surface. The object can slide under the influence of friction forces at each support point. The friction force at point i $(= 1, 2, 3)$ is opposite to the local velocity $\mathbf{v_i}$, and it is proprotional to the normal force N_i: $\mathbf{F_i} = -\gamma \mathbf{N_i v_i}$. The motion is characterized by the center of mass velocity $\mathbf{v(t)}$ and the angular velocity $\omega(t)$. At $t = 0$ the object moves with initial velocity $\mathbf{v_0}$ and angular velocity ω_0.

- **a.** (4 points) First consider the case when the three masses are equal ($m = 1$ kg) and the lengths of the rods are also equal ($l = 0.3$ m), making an equilateral triangle. The initial velocity is $\mathbf{v_0} = \mathbf{0}$, and the angular velocity is $\omega_0 = 3$ rad/s.

  What is the angular velocity at $t = 2$ s? (The damping parameter is $\gamma = 0.07$ s/m; the gravitational acceleration is $g = 9.81$ m/s^2.)

- **b.** (8 points) Consider now the more general case of $m_1 \neq m_2 \neq m_3$ and $l_1 \neq l_2 \neq l_3$. Show that the velocity and angular velocity, respectively, follow $\mathbf{v(t)} = \mathbf{v_0} \exp(-\mathbf{t}/\tau_{\mathbf{lin}})$ and $\omega(t) = \omega_0 \exp(-t/\tau_{rot})$. What is the ratio τ_{lin}/τ_{rot}?

- **c.** (8 points) We add an arbitrary (flat) mass distribution to the system (for example, we make the connecting rods of finite masses). Show that the linear and angular motions remain "independent": i.e., show that the equations of motion separate, and the $\mathbf{v(t)}$ is independent of ω_0, and $\omega(t)$ is independent of $\mathbf{v_0}$.

CM II. (Siegel)

Consider a nonrelativistic particle in three dimensions x, y, z with Hamiltonian

$$H = \frac{(p_x + ay)^2 + p_y^2 + (p_z + bz)^2}{2m}$$

in terms of the conjugate momenta p_x, p_y, p_z and some constants m, a, b.

a. Find the equations of motion.

b. Find the Lagrangian (in terms of $x, y, z; \dot{x}, \dot{y}, \dot{z}$ — no momenta).

c. Solve the equations of motion for x, y, z in terms of the time t and initial conditions.

CM III. (Roček)

Suppose a certain kind of atom radiates with a frequency ν_0 in its rest frame. If we view the atom from another frame, we see a shifted frequency ν.

a. Find the ratio ν/ν_0 if we observe the atom moving away from us with a speed β at an angle θ to the line of sight.

b. For what θ is ν/ν_0 a maximum? A minimum? One?

c. If we observe a cloud of radiating atoms with center of mass at rest and with a fixed speed β uniformly distributed in all directions, what is the average frequency shift that we observe? Expand your result for small β to the leading nontrivial order. From this result, say if the spectral lines from a hot gas are red-shifted, blue-shifted, or at the same frequency relative to the those from a cold gas? (A version of this effect has actually been observed using the Mössbauer effect).

Electricity and Magnetism and Optics

Three problems, work any two.

EM&O I. (Weisberger)

As is shown in the figure, two infinite plane metal sheets of uniform thicknesses t_1 and t_2, respectively, are placed parallel to each other with their adjacent faces separated by a distance d. The space between the sheets is filled with an insulator with electric permittivity ϵ; the space outside is in vacuum. The first sheet has a total charge per unit area equal to σ_1; the second has a total charge per unit area equal to σ_2.

Find the true surface charge densities on each of the four faces, the electric field in all three regions exterior to the plates, and the polarization charge densities on the surfaces of the dielectric.

EM&O II. (Grannis)

A very long concentric pair of cylinders (radii a and b, with $b > a$) separated by vacuum are made from very good conductors. They are connected at one end to a battery with potential difference V and at the other end to a "load" resistor R. Note that this arrangement is like a transmission line (or waveguide), but we have taken the source frequency to be $\omega = 0$.

a. (5 points) Find the electric and magnetic fields for $a < r < b$ in terms of V and R (and constants).

b. (5 points) Using the Maxwell stress tensor or other methods, show that the force per unit area on the outer cylinder is $(B^2/2\mu_0 - \epsilon_0 E^2/2)$ directed radially outward.

c. (5 points) Find the capacitance (C) and inductance (L) per unit length and show that the resistance for which the net force on the cylinders is zero is $R = Z_0$, where $Z_0 = \sqrt{L/C}$ is the characteristic impedence of these same coaxial cylinders viewed as a transmission line.

d. (5 points) Find the total energy/time associated with the static $\vec{E}$ and $\vec{B}$ fields crossing unit area (the Poynting vector) for the zero net force situation. Integrate your result over a suitable area perpendicular to the energy flow and give the interpretation of your result.

EM&O III. (Kirz)

The Mach-Zehnder interferometer consists of two beam splitters (BS) and two mirrors (M), as shown in the sketch. Phase shifts in one of the arms show up as intensity variations in the recombined beams leaving the interferometer. Colella, Overhauser, and Werner (COW) used for neutron beams such an interferometer made from single crystal silicon in 1975 to verify that the wavelength of a monoenergetic neutron beam (NB) is influenced by gravity precisely as one would expect. Start the experiment with the interferometer in the horizontal plane. (Neglect the curvature of the trajectories). Each arm is of length L, and the angle of deflection from one arm to another is 2θ, as is shown in the figure. Now rotate the device around the incident beam by an angle ϕ, so that leg A rises above the incident beam. Find the phase difference between the outgoing beams introduced by the gravitational shift for the given geometry as a function of the incident wavelength. You may assume that the fractional change in wavelength is much smaller than 1. You may also assume that the beam splitters and mirrors are perfectly thin. (In reality this was not a good assumption in the COW experiment.)

FIGURE !!!!!!!!!!!