STATE UNIVERSITY OF NEW YORK AT STONY BROOK
DEPARTMENT OF PHYSICS AND ASTRONOMY

Part I.

Wednesday, 6 September 2000 – Day 2

Comprehensive Examination in Quantum Mechanics and in Statistical Mechanics and Thermodynamics

General instructions: In each of the two areas, do two of the three problems. Each problem should take about $\frac{3}{4}$ hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

Quantum Mechanics

Three problems, work any two.

QM I. (Marx)

A particle of mass m and energy E is moving along the x-axis and encounters a delta function potential barrier at the origin, $V(x) = V\delta(x)$. Calculate transmission probability through the barrier. Explicitly state and defend the boundary conditions you apply.

QM II. (Hemmick)

A particle of mass m moves in the one dimensional potential described by the function below:

$$U(x) = \frac{1}{2}m\omega^2 x^2 + \lambda x^4$$

· **a.** Using the creation and annihilation operators $a^\dagger$, a:

$$a = \left(\frac{m\omega}{2\hbar}\right)^{1/2}(x + \frac{ip}{m\omega}),$$

where p is momentum of the the particle, calculate the energy change to the n^{th} state treating the quartic term as a first order perturbation.

b. Realistically, as n increases, the wave function will spread and the effect of the perturbation will become large. Write an inequality for the principle quantum number n such that the perturbation term provides less than 10% of the POTENTIAL energy for the particle.

QM III. (Brown)

The matrix elements H_{ij} of the Hamiltonian of a three-state system, $i = 1, 2, 3$, $j = 1, 2, 3$, are all equal:

$$H_{ij} = V = \text{constant}.$$

Find the eigenfunctions and energies of this system.

Statistical Mechanics and Thermodynamics

Three problems, work any two.

SM&T I. (Prakash)

- **a.** (10 pts.) Derive an expression for the energy density ε of black body radiation in a cavity of volume V held at temperature T.
- **b.** (5 pts.) Given that the radiant energy flux of a perfect emitter is $(c/4)\varepsilon$, (c is the velocity of light), estimate the total power (energy per second) radiated by your body, neglecting any energy that is returned by your clothes and environment.
- **c.** (5 pts.) The sun's surface temperature is $T_s \cong 6000^o$ K. Which puts out more power per unit mass - the sun or your body? Give a numerical estimate.

Useful relation and constants:

$$\int_0^\infty dx \, \frac{x^3}{e^x - 1} = \frac{\pi^4}{15}$$

$\hbar \cong 1.055 \times 10^{-27}$ erg-sec, $c \cong 3 \times 10^{10}$ cm sec^{-1}, $k_B \cong 1.38 \times 10^{-16}$ erg K^{-1}

$M_\odot \cong 1.989 \times 10^{33}$ g, $R_\odot \cong 6.96 \times 10^{10}$ cm.

SM&T II. (Shafer)

Consider a large number N of identical spin 0 particles of mass m moving non-relativistically in a spherically symmetric potential of the form $V(x, y, z) = \alpha r^3$. The particles are in contact with a heat bath at temperature T.

- **a.** (6 pts.) Consider the case that the system is completely classical. What is the average potential energy of a particle?
- **b.** (7 pts.) In the following, assume that the temperature T is large compared to the level spacings, so that the sum over all levels can be replaced by an integral over phase space. Formulate the condition under which quantum effects in the distribution function have to be taken into account. For this purpose, replace the Boltzmann distribution by the appropriate quantum statistical distribution function and calculate the leading correction to the classical energy.
- **c.** (7 pts.) Determine the dependence of the critical temperature for Bose-Einstein condensation on the parameters of the system, the particle number N, the coefficient α, and the mass m. You do not have to calculate the numerical prefactor.

FIGURE !!!!!!!!!!!!!

An ideal monatomic gas used in a heat engine performs the 4-step cycle shown in the figures above.

a. (3 pts.) During which step(s) is heat absorbed by the gas? During which step(s) is heat released by the gas?

b. (4 pts.) What is the ratio of temperatures during steps 1-2 and 3-4?

c. (4 pts.) Find the work performed during the step 1-2 (expressed in P_0 and V_0).

d. (4 pts.) What is the heat transfer during step 2-3 (expressed in P_0 and V_0)?

e. (5 pts.) Find the efficiency of the cycle and compare it quantitatively to the efficiency of the Carnot cycle.