

STATE UNIVERSITY OF NEW YORK AT STONY BROOK
DEPARTMENT OF PHYSICS AND ASTRONOMY

Part I.

Tuesday, 23 January 2001 – Day 1

**Comprehensive Examination in Classical Mechanics and Special Relativity
and in Electromagnetism and Optics**

General instructions: In each of the two areas, do two of the three problems. Each problem should take about $\frac{3}{4}$ hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

Classical Mechanics and Special Relativity

Three problems, work any two.

CM I. (Stephens)

A solid disk of radius r and mass m rolls without slipping in a circle on the floor, driven by the vertical shaft which rotates with angular speed Ω . The wheel is coupled to the vertical shaft by an axle of length ℓ . Because of its angular momentum, the vertical contact force is greater than the weight mg . Neglect any lateral friction between the wheel and the floor. (Lateral means perpendicular to the plane of the wheel. Some friction is required *along* the path of the wheel to make it roll without slipping, but that does not enter into the problem.)

What is the contact force between the wheel and the floor? Express your result in terms of the variables given above.

leave space for a figure

CM II. (Verbaarschot)

Three particles with identical mass move on a circle with radius R in zero gravity and are connected by identical springs with elasticity properties described by the potential $V(x) = \cosh x$ (where x the length of the spring).

- a. (4 points) Write down the equations of motion and obtain the equilibrium position of the particles.
- b. (4 points) Write down the Lagrangian and expand it to second order about the equilibrium position. Derive the equations of motion and compare them with your result from **a**.
- c. (4 points) Find the normal frequencies and normal modes.
- d. (8 points) Give an interpretation of the normal modes

CM III. (Weisberger)

Consider a spaceship that experiences a constant acceleration of magnitude $g = 9.8 \text{ m/s}^2$ in its rest frame.

- a. What is the speed of the spaceship 1 year after it starts from rest?
- b. How much has an astronaut on the spaceship aged in that time?
- c. How far has the spaceship travelled in that time?

Some Integrals:

$$\int \sqrt{1-x^2} dx = \frac{1}{2}(x\sqrt{1-x^2} + \sin^{-1} x) \quad (1)$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x \quad (2)$$

$$\int \frac{dx}{1-x^2} = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (3)$$

$$\int \frac{dx}{(1-x^2)^{3/2}} = \frac{x}{\sqrt{1-x^2}} \quad (4)$$

Electricity and Magnetism and Optics

Three problems, work any two.

EM&O I. (Goldhaber)

A quarter century before Einstein's $E = Mc^2$, J.J. Thomson found $E = (3/4)Mc^2$. See if you can do that too:

- a. For an electric charge Q uniformly distributed over the surface of a sphere of radius R , compute the electric field $\vec{E}(\vec{r})$ everywhere, the electric field energy density $\mathcal{E}(\vec{r})$, and the total electric field energy. Call this E .
- b. When the sphere moves with constant velocity \vec{v} , compute at first order in \vec{v} the magnetic field $\vec{B}(\vec{r}, t)$ everywhere. Obtain the magnetic field energy density, and integrate this over all space to obtain an expression which is defined as $Mv^2/2$. What is the value of E/Mc^2 ?
- c. Use the results above to compute the momentum \vec{p} in the electromagnetic field, and set $\vec{p} = M\vec{v}$. Compare this with the previous definition of M .
- d. Provide a reason or reasons why this calculation might be off from the Einstein relation.

EM&O II. (Metcalf)

- a. (3 points) A 1 mW beam of monochromatic light is incident perpendicular to a lossless plane mirror of reflection 99%, and the transmitted beam then strikes an ideal power meter. What is the reading on the meter?
- b. (7 points) A second identical mirror is placed a distance L away from, and parallel to, the first one, between it and the power meter. Now the meter reading depends on L . Make a plot of this dependence. Be sure to label both axes **quantitatively**, and explain the character of the plot.
- c. (6 points) For certain values of L the meter reading is higher than in **a.** above. How can this be? If the first mirror reflects 99% of the light, then there's no way the second mirror can recover that lost light and cause it to hit the power meter.
- d. (4 points) If the meter indeed reads higher than the reading of **a.**, then energy conservation appears to be violated, because it seems that the light initially reflected by the first mirror cannot reach the power meter. Explain what really happens to the reflected beam.

EM&O III. (Mihaly)

A magnetic force large enough to compensate for the weight of a ferromagnetic object can be easily achieved, but true “levitation” (stable equilibrium) is a different matter. A ferromagnetic rod in a static magnetic field is always unstable (*Earnshaw’s theorem* states the same for any object of arbitrary, fixed magnetization). Yet levitation by magnetic field has been demonstrated for a long time with superconductors, and researchers in the Nijmegen High Field Magnet Laboratory even reported levitating a living frog!

- a.** (6 points) Assume that there is a magnetic field varying between approximately zero and a peak value of B over a (vertical) distance of approximately $\ell = 0.1\text{m}$. Estimate the required value of B for levitating a frog? (The frog is diamagnetic, with a magnetic susceptibility of $\chi = -10^{-5}$ and density $\rho = 1\text{ g/cm}^3$. Use reasonable estimates for other parameters as needed. $\mu_0 = 1.256 \times 10^{-6}\text{ Tm/A}$)

To investigate the stability of the equilibrium in the simplest possible case, consider a magnetic dipole of moment \mathbf{m} placed in an appropriately created external field of $\mathbf{B}(\mathbf{r})$. Assume that an equilibrium is found. Disregard rotations and characterize the position of the dipole by the displacement, \mathbf{r} , from the equilibrium. For small displacements the change in the magnetic energy can be expressed as $U = \mathbf{r}\mathbf{T}\mathbf{r}$. In a proper system of reference the tensor \mathbf{T} can be made diagonal. The equilibrium position is stable if all three diagonal elements of \mathbf{T} are greater than zero.

- b.** (9 points) Express the elements of \mathbf{T} in terms \mathbf{m} , $\mathbf{B}(\mathbf{r})$, and its derivatives. What is the sum of the diagonal components of \mathbf{T} ? How does this sum relate to Earnshaw’s theorem?
- c.** (5 points) Why is it still possible to have a stable equilibrium for a superconductor or a diamagnet?

Hint: You may want to use the vector identities

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - (\mathbf{A} \cdot \mathbf{B})\mathbf{C} \quad \text{or} \quad \mathbf{m} \times (\nabla \times \mathbf{B}) = \nabla(\mathbf{m} \cdot \mathbf{B}) - (\mathbf{m} \cdot \nabla)\mathbf{B}.$$