STATE UNIVERSITY OF NEW YORK AT STONY BROOK
DEPARTMENT OF PHYSICS AND ASTRONOMY

Part I.

Monday, 27 August 2001 – Day 1

Comprehensive Examination in Classical Mechanics and Special Relativity and in Electromagnetism and Optics

General instructions: In each of the two areas, do two of the three problems. Each problem is designed to take about $\frac{3}{4}$ hour and is worth twenty points. You may take up to four and $\frac{1}{2}$ hours to work the problems, so you have more than enough time if you understand what each problem you are answering is about. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You are allowed to have with you one 8-1/2″ by 11″ page of notes at the exam. It must be readable without magnification, and it will be examined by the exam proctor. The ONLY materials you may have are the examination paper, the exam "bluebooks", your single page of notes, a calculator, and a foreign language dictionary, which will also be examined by the proctor. You may not share any these materials with other students. **No more than four problems will be graded.** You must turn in all bluebooks in which you write anything. For any bluebooks that do not contain solutions to the four problems you want graded, **you must write on the covers of the covers of those bluebooks DO NOT GRADE.**

Classical Mechanics and Special Relativity

Three problems, work any two.

CM I. (Hobbs)

The figure shows a disk of mass M and radius a. It is constrained to roll on the inner edge of a massless hoop of radius R. The hoop is vertical and is forced to rotate about the vertical diameter with an angular velocity ω. The system is in a constant gravitational field of strength g pointing down.

- **a.** (8 points) Find the equations of motion for the disk.
- **b.** (8 points) Find the equilibrium angle of the disk.
- **c.** (4 points) Find the frequency of small oscillations about equilibrium.

HINT: This problem is most easily solved using the Lagrangian method with explicit substitution of equations of constraint. You do not need to use Lagrange multipliers. Do not forget the rotational kinetic energy.

leave room for figure

CM II. (Prakash)

Consider a system of nonrelativistic point particles of mass m_i with position vectors $\mathbf{r}_i$, momenta $\mathbf{p}_i$, and applied forces $\mathbf{F}_i$.

a. (5 points) Show that the total time derivative (d/dt) of the quantity $G(t) = \sum_i \mathbf{p}_i \cdot \mathbf{r}_i$ averaged over a time interval τ may be expressed as

$$\langle 2T \rangle + \left\langle \sum_i \mathbf{F}_i \cdot \mathbf{r}_i \right\rangle = \frac{1}{\tau} \left[G(\tau) - G(0) \right] ,$$

where $\langle \cdots \rangle \equiv \frac{1}{\tau} \int_0^\tau (\cdots)\, dt$ denotes averaging over time t and T is the total kinetic energy of the system. Ignore interparticle interactions.

b. (7 points) Specify at least two physical circumstances in which the so-called virial theorem

$$\langle T \rangle = -\frac{1}{2} \left\langle \sum_i \mathbf{F}_i \cdot \mathbf{r}_i \right\rangle$$

may be established. Derive an expression of this theorem for forces that are derivable from a potential V.

c. (8 points) For a single particle moving under a central force with

$$V(r) = -\frac{a}{r} + br ,$$

consider the case in which the average total energy $\langle E \rangle$ is zero. For this case, find an expression relating the average kinetic energy $\langle T \rangle$ and the average orbit size $\langle r \rangle$.

CM III. (Sprouse)

It is now possible to accelerate radioactive nuclei to high energies. Suppose we can create the radioactive element ^{210}Fr ($t_{1/2} = 3$ minutes) in a nuclear reaction and then accelerate it to high energies.

a. Estimate the rest energy of a ^{210}Fr nucleus in GeV.

b. Estimate the diameter of a ^{210}Fr nucleus in fm (1 fm $= 10^{-15}$ m).

c. A ^{210}Fr nucleus is accelerated in the Relativistic Heavy Ion Collider (RHIC, at nearby Brookhaven National Laboratory) to a kinetic energy of 100 GeV. What is its velocity?

d. How would this energetic ^{210}Fr nucleus appear to an observer in the laboratory frame? Give relevant dimensions in fm, and describe the observed shape.

e. Two samples of ^{210}Fr nuclei are prepared at the same time. One sample is at rest in the laboratory; the other sample is moving with a kinetic energy of 100 GeV. After 3 minutes passes in the laboratory frame, what fraction of each sample remains?

f. Two ^{210}Fr nuclei with equal and opposite momenta are headed toward a collision. If the two nuclei are viewed in the laboratory system when the front of one is at the back of the other, by how much is the density of the nuclei increased over normal nuclear matter density?

Electricity and Magnetism and Optics

Three problems, work any two.

EM&O I. (Siegel)

a. A constant linear charge density λ is located at $x = x_0 > 0$ and $y = y_0 > 0$ for all z. Find the electric potential.

b. Add two intersecting infinite conducting plates, one in the xz plane, the other in the yz plane. Find the electric potential for $x > 0$, $y > 0$ and all z.

EM&O II. (Shrock)

A conducting bar of mass m is attached to, and slides without friction on, two parallel conducting rails. The rails are separated by a width w and connected by a resistance R at the left end, as shown in the figure. The rails and the bar have negligible resistance. A magnetic field B is oriented upward, as shown. At time $t = 0$, the bar is moving to the right at speed v_0.

leave room for figure

a. Is there any induced current in the bar? If you say no, justify your answer. If you say yes, calculate this current including its direction.

b. Determine the position of the bar as a function of time.

c. Determine the position of the bar as a function of time if the resistance is replaced by an inductance L.

EM&O III. (Jacobsen)

In the driven harmonic oscillator model used to describe anomalous dispersion (with damping ignored), the dielectric function is numerically close to ϵ_0, with a value of

$$\epsilon = \epsilon_0 \left(1 + \frac{n_a e^2}{m_e \epsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2} \right) ,$$

where n_a is a volume density of atoms, f_j is a weighting of the response of each of j electron transitions with resonant frequencies ω_j, and ω is the driving frequency. Besides specific atomic transitions, most of the electron transitions ω_j are well into the ultraviolet for solids such as glass.

a. (6 points) Show that for visible light (and no magnetic effects) the refractive index $n = \sqrt{\epsilon/\epsilon_0}$ takes the Cauchy form of

$$n \simeq 1 + A \left(1 + \frac{B}{\lambda^2} \right) .$$

b. (6 points) Show that for X rays (where the radiation frequency is much higher than the frequency of most atomic transitions), the refractive index takes a form $n \simeq 1 - D\lambda^2$.

c. (8 points) Find an expression for the phase $v_p = w/k$ and group $v_g = dw/dk$ velocities of X rays in matter (using $n \simeq 1 - D\lambda^2$). Comment on your results at the level of a sentence each for group and phase velocities.