STATE UNIVERSITY OF NEW YORK AT STONY BROOK
DEPARTMENT OF PHYSICS AND ASTRONOMY
Part I.
Wednesday, 29 August 2001 – Day 1
Comprehensive Examination in "Experiment" and "Breadth"

General instructions: Twelve problems are given. You should do any four, subject to the constraint that you should answer **no more than three** from "experiment" and **no more than three** from "breadth" (that is, not all four problems can be chosen from the same category). Each problem is designed to take about $\frac{3}{4}$ hour and is worth twenty points. You may take up to four and $\frac{1}{2}$ hours to work the problems, so you have more than enough time if you understand what each problem you are answering is about. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You are allowed to have with you one 8-1/2″ by 11″ page of notes at the exam. It must be readable without magnification, and it will be examined by the exam proctor. The ONLY materials you may have are the examination paper, the exam "bluebooks", your single page of notes, a calculator, and a foreign language dictionary, which will also be examined by the proctor. You may not share any these materials with other students. **No more than four problems will be graded.** You must turn in all bluebooks in which you write anything. For any bluebooks that do not contain solutions to the four problems you want graded, **you must write on the covers of the covers of those bluebooks DO NOT GRADE.**

"Experiment"

Expt. I. (Koch)

Do 3 out of 5 of the following questions. In each case state clearly how you do do the measurement to the specified precision and accuracy. In each case include a sketch of the apparatus.

a. Measure to 5% or better the strength and direction of a magnetic field that is uniform over a volume of a few cm^3. The strength is $\mathcal{O}(10^{-3}\ \mathrm{T})$.

b. Measure to 1% or better the "gyromagnetic ratio" (g-factor: ratio of the absolute value of the magnetic moment measured in units of the Bohr magneton $\mu_B = -e\hbar/(2mc)$ to the associated angular momentum measured in units of $\hbar = h/(2\pi)$) for the ground state of the Rb atom.

c. Measure to 10% or better the output power of a diode laser operating at $\lambda \simeq 800$ nm.

d. Measure to 10% or better the temperature T in Kelvin of an alkali vapor (*e.g.,* Rb) inside a closed glass cell. (You cannot "touch" the cell, so you cannot use a " normal thermometer".)

e. Measure the spontaneous radiative lifetime to 10% or better of the first excited "P-state" of an alkali atom, *e.g.,* Rb.

Expt. II. (Aleiner)

The quantum Hall effect (QHE) is observed at low temperatures in a quasi-two-dimensional electron gas, usually created in doped semiconductors. Two Nobel prizes in physics were awarded in this area: in 1985 for the integer QHE (K. von Klitzing) and in 1998 for the fractional QHE (Tsui, Störmer, and Laughlin). The results of a typical measurement are shown in the figure (V.J. Goldman, unpublished).

leave room for figure

a. (5 points) Sketch the sample geometry for measuring the longitudinal resistance, R_{xx}, and transverse resistance, R_{xy}.

b. (9 points) From the experimental plot, find the areal density (measured in units of $1/\mathrm{m}^2$) of the two-dimensional electron gas. The value of the flux quantum is $\Phi_0 = hc/e = 4 \times 10^{-7}$ Tm^2.

c. (4 points) What would happen with R_{xx} and R_{xy} if the magnetic field were flipped: $H \to -H$?

d. (2 points) Which ranges of the magnetic field on the graph correspond to the integer QHE?

Expt. III. (Engelmann)

Describe briefly the following detectors often used in nuclear physics and/or heavy ion collision physics. In each case ake a sketch of the detector element in which the physical process occurs and discuss the physical process responsible for the particle detection. Use this discussion to justify the estimates you are asked to make.

a. (6 points) Scintillation counters: Include plastic scintillators, NaI crystals and photomultipliers. Give and justify estimates of energy resolution and time resolution.

b. (5 points) Solid state counters: Give and justify estimates of energy and time resolution.

c. (3 points) Multiwire proportional chambers: Give and justify estimates of time and spatial resolution.

d. (3 points) Drift chambers: Give and justify estimates of time and spatial resolution.

e. (3 points) Silicon microstrip chambers: Give and justify estimates of time and spatial resolution.

Expt. IV. (Engelmann)

Accurate measurements of the muon "g-factor" open a window to physics beyond the Standard Model of elementary particles. For a pointlike "Dirac" muon theory gives $g = 2$, but for muons in nature g differs slightly from two. (This is usually parameterized as $g_\mu = 2(1 + a_\mu)$, with a_μ called the "muon g-factor *anomaly*".) For longitudinally polarized muons held on a cyclotron orbit by a magnetic field, the deviation from $g = 2$ causes the muon magnetic moment to precess about the magnetic field with a frequency ω_s that differs from the orbital (or "cyclotron") frequency ω_c. Thus, an angle $\phi = (\omega_s - \omega_c)\,t$ between the muon momentum and its spin vector is introduced after a time t. The "anomalous frequency" $\omega_a = (\omega_s - \omega_c) = a_\mu eB/m$, where e = muon charge, m = muon mass, and B = magnetic field; all units are SI. Assume the following values, which are near those used in the muon $(g - 2)$ experiment at Brookhaven National Laboratory: muon energy $E = 3$ GeV, $B = 1.5$ T, and cyclotron radius $r = 7$ m. You may use 0.1 Gev/c^2 and 2 μs for the muon mass and lifetime, respectively.

a. (4 points) Make a sketch of the cyclotron radius, indicate the region of B and its direction, and draw the momentum and spin vectors of the muon after one revolution. Calculate the angle (in degrees) introduced between the muon's momentum and spin vector after each revolution in the cyclotron for $a_\mu \sim 10^{-3}$, as predicted by the Standard Model.

b. (4 points) Describe how the muons are produced and explain the mechanism for obtaining muons with large longitudinal polarization. Make a sketch that illustrates your description.

c. (4 points) Describe how the muon decay angular distribution is used to tag the muon spin direction and, hence, to detect the angle ϕ defined above. Make a sketch that illustrates your description; include the decay process and the detector registering the muon decay particle.

d. (4 points) Using the period you calculate for the anomalous precession and the muon lifetime in the laboratory, make a sketch of the the detected muon decay rate as a function of time (in the laboratory frame) for one muon laboratory lifetime.

e. (4 points) The experimental and theoretical (Standard Model) accuracy for the muon g-factor anomaly is ~ 1 ppm $= 10^{-6}$. The relative error of ω_a and, hence, the relative error in a_μ is ($\frac{1}{A}\times$ relative error from counting statistics), where A is the observed asymmetry in the muon decay; assume $A \sim \frac{1}{3}$. Estimate the number of decays you must observe to reach the accuracy mentioned.

Expt. V. (Wijers)

This problem explores how timing measurements on (almost) periodic sources are used in astronomy for high-precision experiments. A case in point will be radio pulsars, rotating neutron stars that emit and are detected by us one sharp spike of radio radiation per spin period. The rotation phase, ϕ, goes from 0 to 1 in one spin period. For constant spin frequency ν, we have $\phi = \nu t$. In case the spin period varies, we *define* the instantaneous spin frequency as $\nu = \mathrm{d}\phi/\mathrm{d}t$.
To investigate periodic phenomena sensitively, astronomers use the so-called '$O - C$ diagram', or 'observed minus computed diagram'. Here the phase of the observations of peak times for the source signal minus those phases as computed from a model are plotted on the vertical axis, as a function of time.

a. (6 points) Sketch what the $O - C$ diagram looks like for (1) an exact model; (2) a model spin frequency slightly smaller than the true one; (3) a model of a constant frequency, while the true frequency changes linearly with time; (4) a sinusoidially varying source frequency, with a constant model frequency equal to the average source frequency.

b. (8 points) Take a pulsar with a spin period of 10 ms. Let's assume we have an experiment that can measure the pulse peak arrival time to 50 μs per measurement. Assuming a measurement one year after the previous one, to what relative precision can we determine the spin period with this measurement? Note that you need to make the crucial assumption that you have not 'missed a beat', i.e., that you know exactly how many pulses passed since the last measurement. How well do you need to know the pulse frequency at the start of the year to achieve this?

c. (6 points) Pulsar magnetic fields cause them to emit EM waves at the expense of rotational energy. For typical neutron star mass and radius, this leads to $P\dot{P} = 10^{-31}B^2$, with B the magnetic field in Tesla and P the period in seconds. For the same neutron star and instrument as above, what is the smallest magnetic field we can measure in a year, if we know the period with negligible error at the start?

Expt. VI. (Simon)

Each of the following three sections requires an observational solution. After you have identified the technique or techniques you wish to apply, write your answer in the form of a brief summary. For each part, your written answer should be limited to one page of the blue book, and you may use one additional page for one diagram or equation(s). Your text should include the assumptions and uncertainties involved in the measurements proposed. The answers should not be quantitative, but merely describe how to make measurements and the procedure to be followed once measurements are made.

a. You discover a previously unknown Sa galaxy in the Virgo cluster. Its angular diameter is about an arc minute. Describe an observation that will yield its total mass.

b. You discover a globular cluster associated with our galaxy; it had not been noticed before because it lies close to the galactic plane. Describe observations that will yield its distance and age.

c. You discover a neutron star by its X-ray emission. You have reason to believe it is within 100 pc from the Sun. Describe measurements that will yield its distance and luminosity.

“Breadth”

Breadth I. (Orozco)

The decay of an excited (np) state of the hydrogen atom (principal quantum number n and angular momentum quantum number $\ell = 1$) to its $1s$ ground state ($n = 1$, $\ell = 0$) is characterized by an exponential decay with lifetime τ. This lifetime is given by

$$\frac{1}{\tau} = \frac{4}{3}\frac{\omega^3 \alpha}{c^2}\frac{|< J\|r\|J' >|^2}{2J' + 1}, \tag{1}$$

where ω is the angular frequency of the transition, α is the fine structure constant, J and J' are, respectively, the ground and excited state ‘coupled’ angular momenta (made by coupling ℓ and the electronic spin angular momentum s), and $|< J\|r\|J' >|$ is the reduced matrix element between the two states in units of a_0, the Bohr radius.

a. What would be the frequency spectrum of the light emitted by a single atom at rest? Specify the functional form and the spectral full width at half maximum (FWHM).

b. Estimate the characteristic lifetime for the $\lambda = 121.56$ nm transition between the $2p$ and $1s$ states for a single hydrogen atom at rest.

c. Now consider a collection of hydrogen atoms in thermal equilibrium at $T = 1000$ K. Calculate the Doppler shift for an atom at the average speed in the distribution.

d. What would be the frequency spectrum of the light emitted by a gas of excited ($2p$) hydrogen atoms at a temperature of $T = 1000$ K? Give the functional form and the characteristic width of the spectrum.

For your estimates take room temperature to correspond to 1/40 eV and the mass of the hydrogen atom to be 1 GeV/c^2.

Breadth II. (Mendez)

a. (5 points) In the free-electron model, the Fermi energy, E_F, of the conduction electrons in a three-dimensional metal can be written as the product of the numerical constant 0.12 and a function of Planck's constant, the electron mass, and the density of electrons in the metal. Use dimensional analysis (that is, dimensionality arguments) to write an expression for E_F. Alternatively, you may deduce this expression by starting from the definition of the Fermi energy.

b. (5 points) Calculate the Fermi velocity for electrons in copper, knowing that Cu crystalizes in the face-centered cubic system and has a lattice constant of 3.61 Å, and that each Cu atom contributes one conduction electron. (Note: if you could not come up with an answer to part **a.**, use an order-of-magnitude estimate for the Fermi energy of a typical metal.)

c. (5 points) Sketch both the electrical conductivity and the thermal conductivity (electronic contribution only) of Cu as a function of temperature, from room temperature to $T = 0$ K. Using your knowledge of the physical mechanisms and quantities that determine electrical and thermal conductivity, explain in ten sentences or less the reasons that led you to sketch those temperature dependencies.

d. (5 points) The electrical resistivity of Cu at room temperature is 1.70×10^{-6} Ωcm. Calculate the electron mean-free path at this temperature.

Fundamental constants

Electron charge, $|e| = 1.60 \times 10^{-19}$ C; Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J/K; electron mass, $m_e = 9.11 \times 10^{-31}$ kg; Planck's constant, $h = 6.62 \times 10^{-34}$ Js.

Breadth III. (Kuo)

a. Consider a nucleon placed in a shell model potential $V(r) = \frac{1}{2}m\omega^2 r^2 - \frac{2\alpha}{\hbar^2}\hat{l} \cdot \hat{S}$, where m is the nucleon mass, $\hbar\omega = 10$ MeV and $\alpha = 1.2$ MeV. What are the (n, l, j) quantum numbers of the lowest seven orbits? Calculate and tabulate their energies.

b. Let J and T denote, respectively, the total angular momentum and isospin of a nucleus. If the nucleons of nucleus $^{42}_{20}Ca$ are distributed in shell model orbits as $(0s)^4(0p)^{12}(0d)^{20}(1s)^4(0f_{7/2})^2$, derive the J and T values that are allowed for this configuration.

Breadth IV. (Jung)

During the LEP-II run, LEP experiments (ALEPH, DELPHI, L3 and OPAL) were able to observe production of W^+W^- pairs from e^+e^- collisions for the first time. The observation verified the existence of a tri-gauge boson coupling that is predicted in the Standard Model. This allowed the experimenters to make a precise measurement of the W mass.

a. (2 points) What is the minimum required beam energy for this process to occur?

b. (6 points) Draw tree-level (the lowest order) Feynmann diagrams of this process. There are three of them, and two diagrams represent tri-gauge boson couplings.

c. (8 points) The W's decay immediately after their production. What are the possible decay modes and their approximate branching ratios?

d. (4 points) How do you propose to measure the W mass with these data? Qualitatively describe the procedure.

Breadth V. (Yahil)

In this problem we consider a departure from the standard big-bang model and its effect on the big-bang nucleosynthesis of the light elements.

a. What are the major steps in big-bang nucleosynthesis, at what time t after the big bang do they take place, and what are the temperatures T of the universe then? Give a brief summary (a few sentences for each question), not a detailed account.

b. Evaluate the relative numbers and energy densities of the particles that exist in the universe as it enters the nucleosynthesis epoch.

c. In the standard big-bang model, the chemical potentials, μ, of all the above particles are taken to be very small, $|\mu/T| \leq 10^{-9}$ or so. Explain why we are quite sure that this assumption is correct for the electrons and positrons, but have no meaningful experimental or observational limits for the neutrinos.

d. Explain how a small nonzero chemical potential for the electron neutrinos, say $\mu/T \sim 0.1$ would affect big-bang nucleosynthesis. Is the sign of the chemical potential important? Hint: consider equilibrium via reactions of the type $p + e^- \leftrightarrow n + \nu_e$.

Breadth VI. (Solomon)

In an HII region two of the strongest emission lines are from singly ionized oxygen, OII. These transitions have a common lower level corresponding to the $^4S_{3/2}$ state (level 1). The upper levels ($^2D_{5/2}$, level 2, and $^2D_{3/2}$, level 3) are excited by collisions with electrons. Radiative decay produces the emission. These two transitions are important sources of cooling, and their relative intensities are often used to determine the electron density.
Assume that the only transitions are between $^2D_{3/2} - {}^4S_{3/2}$ (*i.e.*, 3–1, denoted λ3726 by its wavelength in Å), and $^2D_{5/2} - {}^4S_{3/2}$ (*i.e.*, 2–1, denoted λ3729 by its wavelength in Å).
The Einstein A coefficients are $A_{2-1} = 3.6 \times 10^{-5}$ s^{-1} for transition 2–1 and $A_{3-1} = 1.8 \times 10^{-4}$ s^{-1} for transition 3–1.
Assume that the downward collision rates are, for 2–1, $q_{2-1} = 1.0 \times 10^{-7}$ cm^3/s, and, for 3–1, $q_{3-1} = 1.0 \times 10^{-7}/$ cm^3/s.

a. (3 points) What is the relation between the upward and downward collision rates?

b. (7 points) Derive (but do not evaluate) a general relationship for the ratio of intensities I(λ3729/λ3726) for the two spectral lines from OII as a function of electron density.

c. (6 points) Obtain and evaluate the ratio in **b.** in the limit of high electron density and in the limit of low electron density. What is the approximate maximum electron density for the low density limit to be valid? What is the minimum density for the high density limit to be valid?

d. (4 points) Why are the OII lines λ3729 and λ3726 called forbidden lines? Why are they relatively strong in many interstellar HII regions but not strong in laboratory situations?