STATE UNIVERSITY OF NEW YORK AT STONY BROOK
DEPARTMENT OF PHYSICS AND ASTRONOMY
Part I.
Monday, 21 January 2002 – Day 1
Comprehensive Examination in Classical Mechanics and Special Relativity and in Electromagnetism and Optics

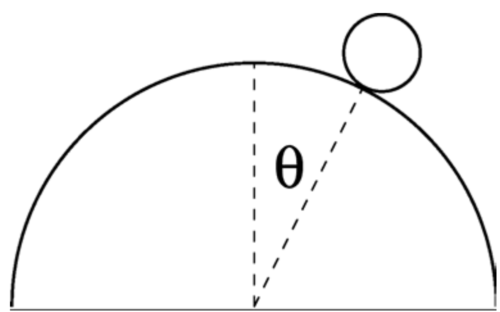
General instructions: In each of the two areas, do two of the three problems. Each problem should take about $\frac{3}{4}$ hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, your one page help sheet, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

Classical Mechanics and Special Relativity

Three problems, work any two.

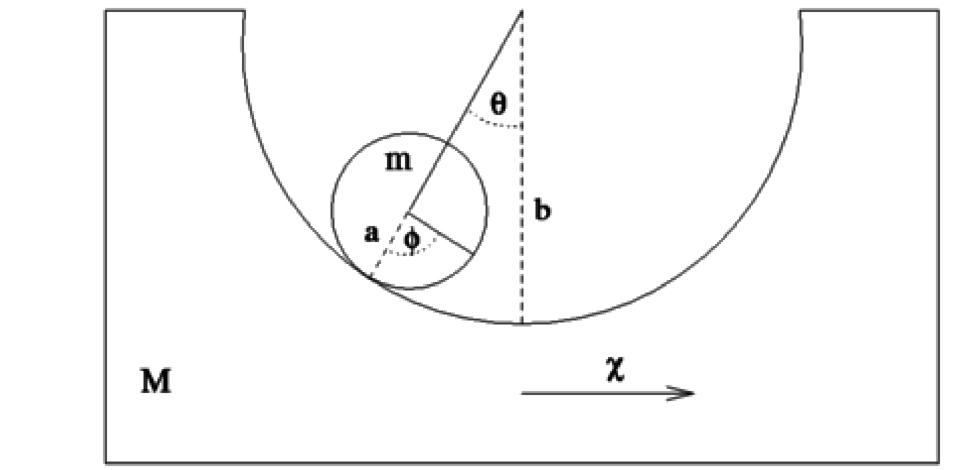
CM I. (Averin)

A sphere of radius r rolls down without slipping on a larger sphere of radius R (see Figure). If the smaller sphere starts from rest on top of the larger one, at what angle Θ does it leave the surface of the larger sphere?



CM II. (Schaefer)

A solid cylinder of radius a and mass m rolls without slipping in a cylindrical groove of radius $b > a$ in a block of mass M, as shown in the figure. The block rests on a frictionless horizontal surface. The moment of inertia of the solid cylinder about the cylinder axis is $I = ma^2/2$.



a. (10 pts) Write down the lagrangian of the system using the generalized coordinates ϕ, θ, χ shown in the figure. Formulate the condition that the cylinder rolls without slipping.

b. (6 pts) Derive the equations of motion in the small-angle approximation. Find the characteristic frequencies.

c. (4 pts) Discuss the normal modes. What happens if the block and the cylinder are initially at rest and then a small impulse is given to the block?

CM III. (van Nieuwenhuizen / Verbaarschot)

We consider a particle with charge e and mass m moving in a plane perpendicular to a constant magnetic field $\vec{B}$ and subject to a harmonic oscillator potential such that the Hamiltonian of the system is given by

$$H = \frac{p_x^2}{2m} + \frac{(p_y - bx)^2}{2m} + \frac{1}{2} m\omega^2 y^2,$$

where $b = e|\vec{B}|/c$. The solution of the Hamiltonian equations for (1) can be obtained more easily if we first apply a canonical transformation to the coordinates and the momenta.

a. (4 points) Consider the transformation

$$x = X, \qquad p_x = P_X, \tag{1}$$

$$y = -P_Y, \qquad p_y = Y. \tag{2}$$

Show that this transformation is canonical.

b. (4 points) What is the Hamiltonian in terms of the new variables? What are the new Hamilton equations of motion?

c. (7 points) Solve the equations of motion for X and Y.

d. (5 points) Construct the general solution for the Hamiltonian in (1). Check that it has the correct number of arbitrary parameters.

Electricity and Magnetism and Optics

Three problems, work any two.

EM&O I. (Goldhaber)

a. (5 points) Show how to superpose in the same region of space static electric and magnetic fields that, acting together, will *not* deflect a beam of charged particles having a given charge q and a particular velocity $\vec{v}$. Such a device is a velocity selector.

b. (15 points) A beam of indistinguishable, minimum-ionizing (i.e., electric charge $q = \pm e$) particles is put through a velocity selector with $E_y = cB_z \times 0.995$. Particles emerging from the velocity selector in the $+x$ direction enter a region of uniform magnetic field $B_z = 10.1$ T, and move in a circular arc of radius 6.2 m, curving in the $+y$ direction. What are these particles?

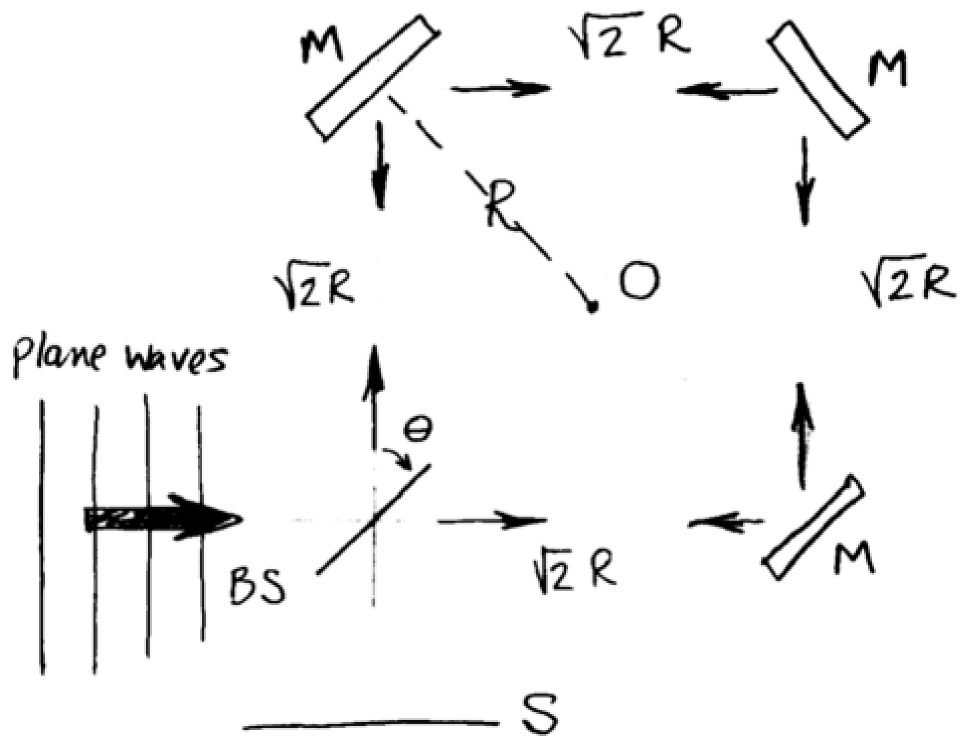
Data: Conventions – right hand rule for electromagnetic fields and forces, charge of electron $= -e = -1.6 \times 10^{-19}$ C. Electron mass $m_e = 9.1 \times 10^{-31}$ kg. Proton mass $m_p = 1836\ m_e$. Nuclear mass, for mass- or nucleon-number A, is within 1% of Am_p. Speed of light is $c = 3 \times 10^8$ m/s.

EM&O II. (Schaefer)

Consider a dielectric sphere (dielectric constant ϵ) of radius a. The sphere is immersed in a static electric field which at large distance is given by $E_0\hat{z}$. Calculate $\vec{D}$ and $\vec{E}$ inside the sphere.

EM&O III. (Koch)

The figure shows a planar arrangement of three mirrors (M) and one beam splitter (BS) located at the corners of a square with side length $2\sqrt{R} = 1$ m. A light source sends a plane-wave beam of monochromatic electromagnetic waves of wavelength $\lambda = 650$ nm at the BS in the lower left corner; the beam is pointed along the bottom side of the square. A viewing screen (S) is placed "below" the BS, along a line of sight that is collinear with the left side of the square and perpendicular to S. Assume that the BS has negligible thickness and that at each M the reflection is off the front surface. Assume that the BS is very close to, but not exactly at, 45 degrees to the side of the square.



a. (6 points) For the arrangement described above, what does an observer see on the screen S? A qualitative answer is ok, but you must justify your answer.

b. (8 points) Let the entire device (source + optical elements + viewing screen) be rotated about an axis through O and perpendicular to the plane at a slow rate, $\omega R \ll c$, where c is the speed of light. For numerical purposes take ω to correspond to 4 rev/s. Does this rotation cause any change in what an observer "sees" on the screen compared to part **a**? Be specific and quantitative in your answer. Replace the light beam with a source of atom waves and the optical elements BS, M, and S with "atom optical" elements: each BS is now an atom-wave beam splitter, each M is now an atom-wave mirror, and the screen S is now an atom-detecting surface. (Do NOT worry about how these devices work. They do exist, but how they work is not your concern in this problem.) Use as your "atom waves" ^{85}Rb atoms traveling at $v = 100$ m/s; it may be helpful to know that 1 amu $= 1.66 \times 10^{-27}$ kg and $h = 6.63 \times 10^{-34}$ Js.

c. (6 points) Given the changes noted in the previous paragraph, what is now the answer to part **b**?