STATE UNIVERSITY OF NEW YORK AT STONY BROOK
DEPARTMENT OF PHYSICS AND ASTRONOMY
Part II.
Wednesday, 23 January 2002 – Day 3
Comprehensive Examination in "Experiment" and "Breadth"

General instructions: Twelve problems are given. You should do any four, subject to the constraint that you should answer **no more than three** from "experiment" and **no more than three** from "breadth" (that is, not all four problems can be chosen from the same category). Each problem should take about $\frac{3}{4}$ hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, your one page help sheet, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

"Experiment"

Experiment I. (Schaefer)

At GSI in germany positively charged kaons are produced in collisions of protons according to $p+p \to K^+ + \Lambda + p$. The Λ particle is a neutral baryon resonance. The masses of the particles involved are $m_p c^2 = 935$ MeV, $m_{K^+} c^2 = 495$ MeV and $m_\Lambda c^2 = 1116$ MeV. (Note: $\hbar c = 200$ MeV fm.)

a. (5points) Explain why the reaction $p + p \to K^+ + n + p$ (n is a neutron) is not observed.

b. (5points) The experiment is performed with a beam of protons hitting a hydrogen target. What is the minimum proton energy E required to produce kaons?

c. (10points) The experiment is repeated with a ^{197}Au target. The dominant production mechanism for kaons is the collision of projectile protons with individual protons in the target. It is nevertheless observed that the threshold energy is lower as compared to the experiment with the hydrogen target. This is related to the fact that a ^{197}Au nucleus is a system of fermions, so the protons and neutrons in the nucleus cannot all be in the same state. Use the measured ^{197}Au radius $r_{Au} = 6.4$ fm to compute the maximum proton momentum p_F in a Au nucleus. Use your result to estimate the minimum proton energy required to produce kaons on ^{197}Au. You can assume that $p_F \ll m_p$.

Experiment II. (Orozco)

Along the way to the work that eventually led to his sharing the 2001 Nobel Prize in physics, Wolfgang Ketterle showed a way to increase the density of atoms in an optical trap by "shelving" them in a state where they do not interact with the laser light that is doing the cooling. [This was published in "High densities of cold atoms in a dark spontaneous-force optical trap," W. Ketterle, K.B. Davis, M.A. Joffe, A. Martin, and D.E. Pritchard, Phys. Rev. Lett. **70**, 2253-2256 (1993).] The experiment was done with ^{23}Na; its ground state is $3s\,^2S_{1/2}$, its nuclear spin is $I = 3/2$, and its ground-state hyperfine splitting is 1.77 GHz.

a. Make an energy level diagram of the electronic ground state. Explain the origin of the quantum number, typically called F, with z-projection M_F, used to label each level, and give the values of F and M_F for each level in your diagram. (Assume that a weak static magnetic field provides the quantization axis.)

b. For a sample in thermal equilibrium at room temperature, calculate what fraction of the population of the $3s\,^2S_{1/2}$ ground state has the largest value of F.

c. The D_2 "line" connects the $3s\,^2S_{1/2}$ ground state with the $3p\,^2P_{3/2}$ excited state. Make an energy level diagram of the $3p\,^2P_{3/2}$ state; label each level with its quantum numbers F and M_F.

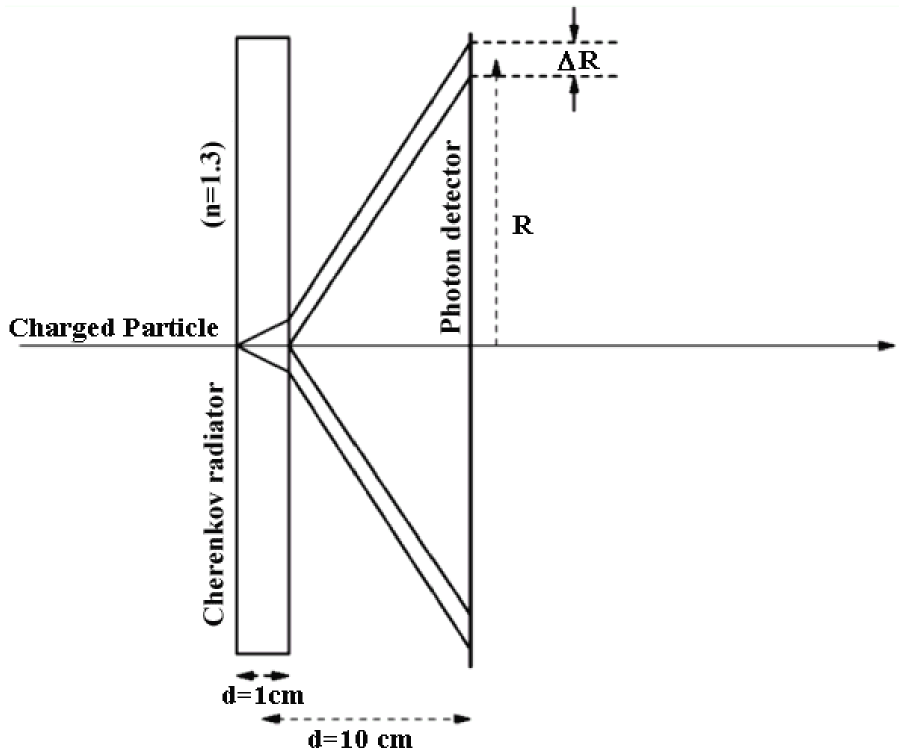
d. For electric dipole transitions involving the D_2 "line", state the appropriate selection rules for changes in quantum numbers F and M_F. Assume the direction of propagation of the laser is parallel to the weak magnetic field. State explicitly the polarization of the light.

e. You have a laser that is tunable around the D_2 "line" and whose polarization can be controlled. The laser beam can be directed onto a sample of very cold ($T < 1$ mK) ^{23}Na with all the population in the highest F value of the ground state. Describe a scheme using the laser whose end result would be to move the population of the ^{23}Na atoms to the smallest F-value in the ground state. Be sure to state what specific levels (F, M_F) in the $3s\,^2S_{1/2}$ and $3p\,^2P_{3/2}$ states you are connecting, what is the quantization axis, and what is the polarization of the laser.

Experiment III. (Hemmick)

Many scattering experiments require that the particle type of a produced particle be identified uniquely. One method of particle identification is to measure both the energy (or momentum) and velocity of a particle and then calculate its mass. Such an identification becomes increasingly difficult at the highest energies since the velocity cannot exceed c.

The device sketched below is a "proximity-focussing" Cherenkov detector. Cherenkov light generated in the radiator section is eventually deposited upon the photon detector in a ring-shaped region whose radius determines the velocity of the particle. Throughout this problem we shall assume that particles always enter the detector at normal incidence.



a. *(4 points)* Derive the angle of the Cherenkov radiation (in the radiator) as a function of the velocity, β, of the particle. (HINT: Cherenkov radiation is the optical analogy to the sonic boom).

b. *(6 points)* Using your result from part a, calculate the mean radius, R, and the width, ΔR of the band which that contains all the photons. Use the approximation $D >> d$.

c. *(6 points)* Assume that a single ring contains 20 detected photons and that the photon detector's spatial resolution is $<< \Delta R$. Determine the velocity resolution $\delta\beta$ when $n\beta = 1.1$.

d. *(4 points)* Compare this performance with a conventional "Time-of-Flight" detector having a time resolution of 100 psec and a flight path of one meter. Give a counter-example for which the conventional detector is a superior choice.

Experiment IV. (Graf)

List three different experimental methods of measuring low (i.e., <100K) temperatures. For each method, discuss
- the device or material used
- the physical property measured
- an equation or a sketch relating the physical property to the temperature (you do not need to derive the relationship)
- the temperature range over wich the device is useful
- the experimental procedure by which the tempeature is determined, specifically showing diagrams of the apparatus.

Experiment V. (Walter)

Eclipsing binaries can be used not only to determine masses and radii, but also to obtain distances. Consider a totally eclipsing pair of O stars in the Large Magellanic Cloud. Their temperatures are determined spectroscopically to each be 50,000 K. Assume that the stars have nearly blackbody spectra. The 2 stars can be assumed to have identical masses and radii. Some observed parameters of the system are:
Maximum apparent visual brightness in V band (wavelength at center of V band is $\lambda_V = 5400$ Å): $V_{max} = 11.25$
Stellar orbital velocities: $v_1 = v_2 = 150$ km/s
Eclipse duration (first→fourth contact): 2.16 days
Some other useful information:
Visual flux in V band: $f_V = 3.67 \times 10^{-9} 2.512^{-V}$ erg/cm^2/s/Å
Blackbody (astrophysical) flux at wavelength λ: $f_{BB} = \pi \frac{2hc^2}{\lambda^3} \frac{1}{e^{hc/\lambda kT} - 1}$ erg/cm^2/s/Å

a. (*16 points, with partial credit given*) Calculate the distance to the binary (and, thus, to the Large Magellanic Cloud).

b. (*4 points*) Suggest two other techniques of measuring the distance to the Large Magellanic Cloud.

Experiment VI. (Lanzetta)

A deep image of the sky, of angular extent 5 arcmin on a side, reveals 15 galaxies of redshift $2.9 < z < 3.1$ and of average apparent magnitude $AB(I) = 24.5$ (i.e., of average apparent magnitude $AB = 24.5$ at observed-frame $I-$band wavelength, or roughly 8000 Å).

a. (*7 points*) Show that in an Einstein–de Sitter cosmological model ($\Omega_M = 1, \Omega_\Lambda = 0$) the average luminosity of the galaxies is $L_\nu = 1.9 \times 10^{28} h^{-2}$ erg s^{-1} Hz^{-1}, where $h = H_0/100$ km s^{-1} Mpc^{-1} is the dimensionless Hubble constant. Note that in an Einstein–de Sitter cosmological model the relationship between luminosity distance d_{lum} and redshift is

$$d_{lum} = \frac{cz}{H_0} \frac{(1+z)^{1/2} + 1 + z}{(1+z)^{1/2} + 1 + z/2} ,$$

where c is the speed of light. Note that $AB = 23.9$ corresponds to energy flux density 10^{-29} erg s^{-1} cm^{-2} Hz^{-1}. Also note that in an expanding universe, the relationship between energy flux and luminosity involves the distance d_{lum}.

b. (*7 points*)Show that in an Einstein–de Sitter cosmological model the comoving volume over which the galaxies are distributed is $\sim 4290 h^{-1}$ Mpc^3. Use the fact that in an Einstein–de Sitter cosmological model the relationship between cosmic time t and redshift z is

$$t = \frac{1}{H_0}(1+z)^{-3/2} .$$

Finally note that in any cosmological model the relationship between angular diameter distance d_{ang} and d_{lum} is

$$d_{ang} = d_{lum}(1+z)^{-2} .$$

Hint: Work out the volume bounded by a rectangular box of side $d_{ang}\theta$ and depth $c\Delta t$, and recall that comoving distances are stretched a factor $1 + z$ with respect to proper distances.

c. (*6 points*) Determine the comoving luminosity density (and its uncertainty) contributed by the galaxies to the universe at rest-frame wavelength $\approx$ 2000 Å (in units, say, of erg s^{-1} Hz^{-1} Mpc^{-3}).

"Breadth"

Breadth I. (van Nieuwenhuizen)

A Colloquium talk about the recent BNL measurement of the g factor of the muon discussed the change in the precession of the spin, caused by the center-of-mass motion. We will consider the spin as a classical angular momentum, and characterize the precession in a magnetic field $\vec{B}$ by the angular frequency $\vec{\omega}$ measured in the particle's inertial (rest) frame. (Here e is the electric charge and m is the mass of the particle. CGS units are used, but you are free to work with other units.)

a. (4 points) Write down the equation of motion for the spin $\vec{s}$ of a particle with magnetic moment $\vec{\mu} = \frac{e}{2mc} g \vec{s}$, at rest in a constant magnetic field. Check the dimensions of this equation. What is the (approximate) value of g for a muon? What is the Lorentz force on a particle that is moving relativistically? Define all symbols.

b. (5 points) A positively charged muon with energy $E = 3GeV$ and mass $105 MeV/c^2$ moves on a circular orbit in the xy plane under the influence of a constant homogeneous magnetic field B of 10^4 Gauss (1 tesla) which points in the positive z direction. What is the radius R of this circle? (Neglect synchrotron radiation). Given that the muon lifetime is about 2×10^{-6} seconds, how many revolutions does a muon make on the average before it decays?

c. (5 points) If in addition to the magnetic field $\vec{B}$ a small electric field $\vec{E}$ is applied to the muon, which is orthogonal to $\vec{B}$ and points radially outward, the muon sees in its inertial frame an induced magnetic field $\vec{B}'$. Determine $\vec{B}'$. How much will the precession frequency change?

d. (6 points) The muon's inertial frame rotates with respect to the laboratory frame with an angular velocity

$$\vec{\omega}_T = \frac{\gamma^2}{\gamma + 1} \frac{\vec{a} \times \vec{v}}{c^2}$$

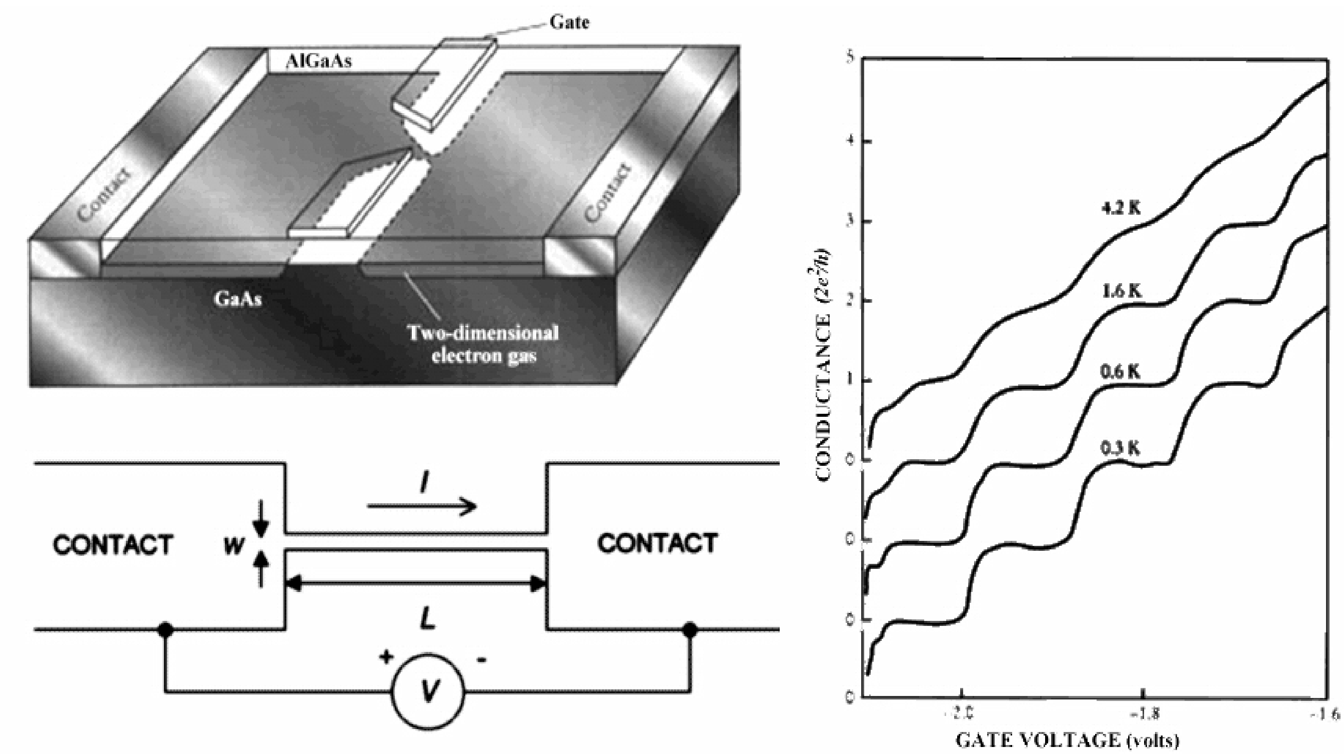
where $\vec{a}$ is the acceleration of the muon in the laboratory frame (the Thomas precession). As is well-known, the Thomas precession reduces the spin-orbit coupling energy of a hydrogen atom by a factor 2. We now study the application of the same idea to relativistic muons. Show that the Thomas precession due to the contribution of $\vec{E}$ to $\vec{a}$ largely cancels the precession of the muon spin due to $\vec{B}'$. (Exact cancellation can be achieved by choosing the muon energy carefully).

Breadth II. (Likharev)

In 1988, two European groups independently discovered the low-temperature effect of quantization of the electric conductance $G = I/V$:

$$G = n(2e^2/h) \tag{1}$$

of submicron-size "ballistic channels" formed in semiconductor structures with 2D electron gas. Conceptually, the contact may be considered as a narrow 2D channel of length $L >> W$, connecting two broader parts ("contacts") - see the Figure. The integer n in Eq. (1) changes by one each time when the channel width W (regulated with negative voltage applied to special "gate" electrodes in the experiment, as shown in the Figure) reaches one of certain values W_n.



a. (5 points) Explain qualitatively the effect described by Eq. (1). Express the values W_n via any parameters of the system you find necessary to introduce (e.g., Fermi energy of the 2D electron gas, electron mass, etc.).

b. (10 points) Derive Eq. (1). Explain why G does not depend on the length and width of the channel. When a voltage V is applied to the channel, the current $I = GV$ flows and electric the power $P = IV$ is dissipated. Where do you think this dissipation is localized?

c. (5 points) Estimate the maximum temperature at which this effect may be observed in GaAs system (effective electron mass $m = 0.067m_0$), for the channel width $W = 0.1$ mm. How large may be the voltage V applied to the channel? How uniform should be the potential energy distribution along the channel? What other important experimental conditions can you foresee?

The potential walls confining the channel may be assumed infinitely high. Scattering of electrons (e.g., by impurities or phonons) and their Coulomb interaction inside the channel may be neglected. Reflection of the electrons from the sharp edges of the channel may be also ignored (because in practice the edges are smooth).

Breadth III. (Dawson)

The Fermilab Tevatron has recently been upgraded to have $p\overline{p}$ collisions at a center-of-mass energy of $E_{CM} = 2\ TeV = 2000\ GeV$. One of the primary goals of the machine is to find the spin-0 Higgs boson predicted by the Standard Model of particle interactions.

a. Describe why the observation of the Higgs boson is important to verify the correctness of the Standard Model of particle interactions. In particular, which specific problem of the Standard Model does the introduction of the Higgs boson solve? (Answer in one brief paragraph).

b. The Higgs boson decays promptly after it is produced. For a Higgs boson of mass 120 GeV/c^2, state at least two allowed decays. If the Higgs mass is 200 GeV/c^2, additional decay modes become available. State at least two of these additional modes.

c. If the Higgs boson is observed at the Tevatron, it will probably be through a $q\overline{q}$ annihilation process that produces both a W and a Higgs. Draw the Feynman diagram for this process and explain what the experimental signature for this process is, (*ie*, which decays of the W and the Higgs will be the likeliest to be observed).

d. Write an expression for the cross section for the Feynman diagram in part c. Specify its dependence on the coupling constants but ignore dimensionless constants of order unity (*e.g.* π).

Breadth IV. (Metcalf)

The Bohr formula (Bohr, 1913) for the energies of the states of atomic hydrogen (H) is

$$E_n = -mc^2\alpha^2/2n^2, \tag{2}$$

where m is the electron rest mass, $\alpha = e^2/\hbar c \approx 1/137 \approx 7.3 \times 10^{-3}$ is the Sommerfeld fine structure constant, and n is an integer ≥ 1.

a. (4 points) Equation (1) is easily derived from an expression involving the classical force on an electron in the Coulomb field of the proton, plus one additional condition. State what is that additional condition, and use it to derive Eq. (1).

b. (4 points) Using the equation he discovered, Schrödinger (1925) treated the H atom and obtained the same formula as Eq. (1). However his mathematical path to the energy levels requires separation of the spatial variables of a three-dimensional partial differential equation. This is usually done in spherical coordinates; the angular parts introduce two new quantum numbers in addition to the (principal quantum number) n of the Bohr energy formula. What are these two new quantum numbers, and what values may they have? State your answer in a few lines. A detailed or mathematical derivation is not needed.

c. (4 points) Whether Bohr's derivation or the Schrödinger equation is used to obtain Eq. (1), often only the motion of the electron is considered, with the argument that the nucleus is so massive that it can be neglected. Show how to justify this approximation. When the contribution of the nuclear motion **is** included, what is the usual form of the correction to Eq. (1)? (Note: This is **not** the overall motion of the atom in the laboratory coordinates.)

d. (4 points) The Schrödinger equation is non-relativistic, but the Dirac equation is fully relativistic. Corrections to the Bohr/Schrödinger non-relativistic energy levels can be found from the Dirac equation. The overall result is usually called *fine structure*. In no more than a few sentences each, describe the physical origin of **two** contributions to fine structure shifts of the energy levels of the H atom. Outline the steps needed to calculate each contribution you discuss, but note that a detailed or mathematical derivation is **not** needed.

e. (4 points) Draw three different energy level diagrams for the $n = 1, 2,$ and 3 energy states of the H atom. In each diagram use all appropriate labels for each level shown. In the first diagram, show only the Bohr levels; give the energies numerically in some convenient units (state what they are!). In the second diagram, show the effect of the two new quantum numbers introduced in (b). In the third diagram, include the fine structure. Because a drawing made to sacle will not be able to show all of them, be sure to give the approximate magnitudes of all energy spacings on your diagram.

Breadth V. (Yahil)

a. State and prove Olbers paradox for an infinite, static, homogeneous, and time-independent universe.

b. Explain how the paradox is removed if the universe has existed for only a finite amount of time, irrespective of its expansion.

c. Give a physical explanation for the correct relativistic expression for the extragalactic component of the surface brightness of the sky, which can be written as an integral over cosmic time:

$$S_0 = \frac{c}{4\pi} \int_0^{t_0} (R(t)/R_0)^4 j(t) \mathrm{d}t \tag{3}$$

where c is the speed of light, $R(t)$ is the scale factor of the universe, $j(t)$ is mean emissivity due to galaxies, and the subscript 0 denotes the present epoch.

d. Work out the expected brightness due to nonevolving galaxies which have a constant comoving density n_0 and a mean luminosity L_0 in a flat, matter universe ($\Omega = 1$, $\Lambda = 0$).

Breadth VI. (Solomon)

This question concerns interstellar reddening and extinction.

a. (*2 points*) What is interstellar reddening?

b. (*2 points*) What is color excess?

c. (*4 points*) Describe how, typically, interstellar extinction varies with wavelength. Explain how the total interstellar extinction can be estimated from the color excess.

d. (*6 points*) Assume that all interstellar grains have a radius of about 150 nm (1500 Å). If interstellar grains were ten times larger in radius, how would this affect the total extinction and the color excess? Assume that the total mass and internal density of interstellar grains are the same. Interpret this result. Could there eexist a substantial amount of very large grains?

e. (*6 points*) The typical interstellar radiation field due to starlight in the galaxy can be represented approximately by a blackbody at 10,000 K, but with a geometrical dilution factor of 10^{-14}. Calculate the temperature of an object which radiates like a blackbody in radiative equilibrium with the starlight. Is this the correct temperature for interstellar grains? If not, why not? Estimate the correct temperature by correcting approximately for the blackbody model.