STONY BROOK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY

Part I.

Tuesday, 3 September 2002 – Day 1

Comprehensive Examination in Classical Mechanics and Special Relativity and in Electromagnetism and Optics

General instructions: In each of the two areas, do two of the three problems. Each problem should take about $\frac{3}{4}$ hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, your one page help sheet, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

Classical Mechanics and Special Relativity

Three problems, work any two.

CM I. (Abanov)

A bead of mass m is put in the middle of a horizontal massless string of length L and tension $F_T \gg mg$.

a. (8 points) What is the vertical displacement of the bead in the equilibrium position?

b. (12 points) If two beads of mass m each are released at a distance $l \ll L$ from each other in the middle of the string they start moving towards each other. In what time will they collide? (Neglect all vertical oscillations of the string.)

CM II. (Weisberger)

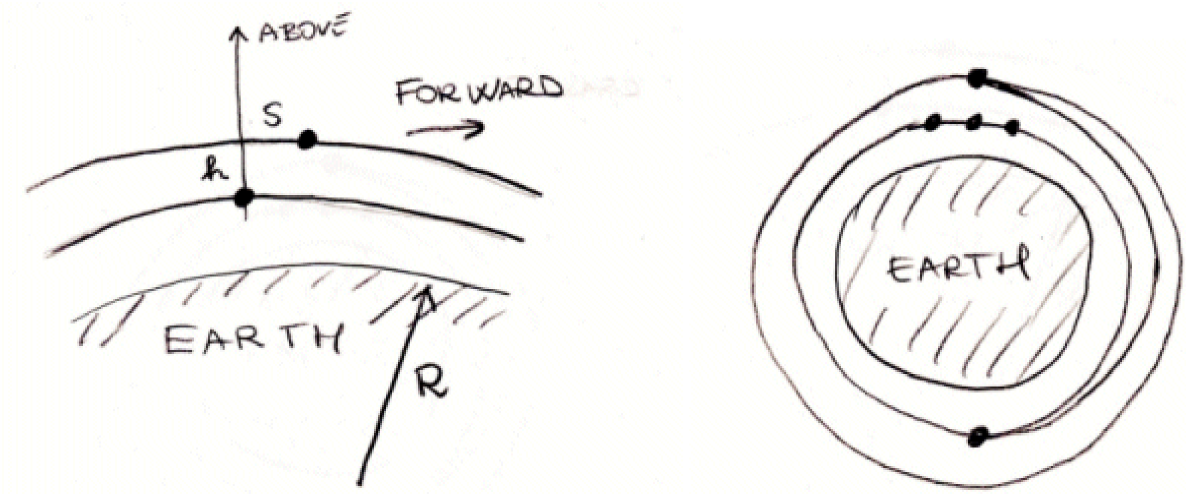
A wedge shaped block with mass M and base angle θ sits on a horizontal frictionless table. A block of mass m is free to slide without friction down the inclined side of the wedge.

a. (15pts) The block of mass m is released with the system initially at rest. Find the displacement of block M as a function of time as the other block slides down its side.

b. (5 pts) For what value of the base angle will the block M move the farthest in a given time?

CM III. (Mihaly)

Spacecrafts A and B are orbiting around the Earth on circular orbits at a few hundred kilometers above sea level. At time $t = 0$ spacecraft B is near and above spacecraft A, and its relative position is given by the parameters s and h defined in the Figure. Assume that $s << R$ and $h << R$, where R =6.6 x 10^3km is the distance to the center of the Earth. The period of the motion of spacecraft A is T = 91minutes. Neglect the gravitation between the spacecrafts, and the effect of the Sun, Moon or any other objects.



a. (5 points) Assume that spacecraft B is exactly above spacecraft A, at $s = 0$, $h = 1.2$km. What will be the relative position of the two spacecrafts at the end of the first full period , $t = T$? Give your answers in terms of s, and specify if it is forward, behind or exactly above.

b. (8 points) This time spacecraft B is also at $h = 1.2$km, not exactly above, but at some position $s \neq 0$. The two spacecrafts want to get connected. In order to achieve this, the astronaut in spacecraft B can operate the propulsion system in a single burst, causing a sudden increase or decrease of the magnitude of the velocity (but the direction of the velocity does not change).

After firing the rockets of spacecraft B, the orbits of the two spacecraft meet at the point hafway in the (circular) orbit of spacecraft A. Was the velocity increased or decreased? How big was the change?

c. (7 pts) What should s be so that the two spacecrafts meet?

Electricity and Magnetism and Optics

Three problems, work any two.

EM&O I. (Goldhaber)

a. (5 pts.) A uniform spherical shell of charge (total charge q, radius a) is in an insulating medium with dielectric constant relative to vacuum $\epsilon > 1$. Compute the electric energy of this system.

b. (5 pts.) Now assume that the half-space $x > 0$ is vacuum, while the half-space $x < 0$ has dielectric constant ϵ. Using the result in **a.**, infer the direction of the force on the sphere for an arbitrary point $\vec{r}$, where $\vec{r} = (x, y, z)$ is the position of the center of the sphere.

c. (10 pts.) Using the method of images, determine the electric field due to the image charge for any $\vec{r}$ with $|x| > a$. Compare with the inference in **b.**

EM&O II. (Roček)

Consider a charge e with rest mass m, initially at rest at the origin. It is acted on by a constant electric field E in the postive y-direction and a constant magnetic field B in the positive z-direction. Work in natural units ($c = 1$).

a. (5pts) If $|E| < |B|$, find the Lorentz transformation to a frame that simplifies the problem. In this frame, the trajectory is circular. What is the initial velocity of the particle?

b. (7pts) Find the frequency and amplitude of the motion in this frame.

c. (3pts) Now transform back to the lab frame. What is the direction and magnitude of the average velocity in this frame?

d. (5pts) Now consider $|B| < |E|$. Find the Lorentz transformation to a frame in which this problem is simple. What is the qualitative behavior of the solution in the lab frame?

EM&O III. (Jacobsen)

In matrix methods of describing propagation of light rays, we consider how a starting angle α_0 and height above the optical axis y_0 is transferred to a final angle α_1 and height y_1. For a ray traveling a distance L in free space, the translation matrix is

$$\begin{bmatrix} \alpha_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ L & 1 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ y_0 \end{bmatrix} \tag{1}$$

Using Snell's law in the small α and small y approximation, refraction at a single optical interface with radius of curvature R can be described by

$$\begin{bmatrix} \alpha_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \frac{n_0}{n_1} & \frac{1}{R}\left(\frac{n_0}{n_1} - 1\right) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ y_0 \end{bmatrix}. \tag{2}$$

The thin lens can be described by

$$\begin{bmatrix} \alpha_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ y_0 \end{bmatrix}. \tag{3}$$

Problem:

a. (8 points) Derive Eq. 2.

b. (6 points) Derive the expression for the thin lens focal length f in Eq. 3.

c. (6 points) Determine the equivalent focal length for two $f = +10$ cm lenses placed 5 cm apart.