STONY BROOK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY
Part II.
Thursday, 5 September 2002 – Day 3
Comprehensive Examination in "Experiment" and "Breadth"

General instructions: Twelve problems are given. You should do any four, subject to the constraint that you should answer **no more than three** from "experiment" and **no more than three** from "breadth" (that is, not all four problems can be chosen from the same category). Each problem should take about $\frac{3}{4}$ hour and is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, your one page help sheet, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

"Experiment"

Experiment I. (McGrew)

Recent atmospheric neutrino results from Super-Kamiokande are consistent with neutrino oscillations between atmospheric muon neutrinos and some unknown neutrino flavor (possibly the tau neutrino). In neutrino oscillations, neutrinos are created in an eigenstate of the weak interaction, ($|\nu_e\rangle$, $|\nu_\mu\rangle$, or $|\nu_\tau\rangle$), but propagate according to their mass eigenstates ($|\nu_1\rangle$, $|\nu_2\rangle$, $|\nu_3\rangle$).
The Super-Kamiokande data is well fit by oscillations between two neutrino families which results in a muon neutrino survival probability of

$$P_{\nu_\mu \to \nu_\mu}(L) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}. \tag{1}$$

a. Derive the above expression assuming two component oscillations between the weak interaction eigenstates ν_μ and ν_τ assuming the mass eigenstates are ν_1 and ν_2 Please use bra-ket notation in units where $\hbar = c = 1$. The following approximation (valid when $y \ll x$) may be useful:

$$\sqrt{x^2 + y^2} \simeq x + \frac{y^2}{2x} \tag{2}$$

b. Neutrinos have very small interactions and are thereby difficult to detect. Discuss the techniques by which Super-Kamiokande and other water tank detectors measure neutrinos. Include in your discussion how the neutrinos interact with the water, how this interaction is subsequently detected, and how the direction of the incoming neutrino can be deduced.

Experiment II. (Drees)

a. Electrons and positrons colliding at beam energies of 100 GeV can annihilate into quark-antiquark pairs. These quarks will fragment into jets of hadrons. Assume that the produced quarks are of u or d flavor:

- Draw a Feynman diagram of the collision process.
- What is the invariant mass of the two jets?
- Relate the four-momentum transfer, Q^2 to the beam energy, and to the angle at which the quark jet is emitted (measured with respect to the beam axis).

b. In proton-proton collisions at the same beam energy, scattering processes with large momentum transfer, Q^2, between quarks from both beams will also produce jets.

- Describe the basic differences of the jet kinematics produced in a parton-parton scattering process as compared to electron-positron annihilation.
- What is the invariant mass of the two jets in terms of the momentum fractions, x_1 and x_2, of the quarks.
- Sketch the probability, as a function of x, to find a quark with momentum fraction x inside a proton.

c. In many experiments, the jet energy is measured by a hadronic calorimeter.

- Explain the basic principle of a hadronic calorimeter.
- Why is the fractional energy resolution ($\frac{\delta E}{E}$), of a calorimeter measurement inversely proportional to the square root of the jet energy?

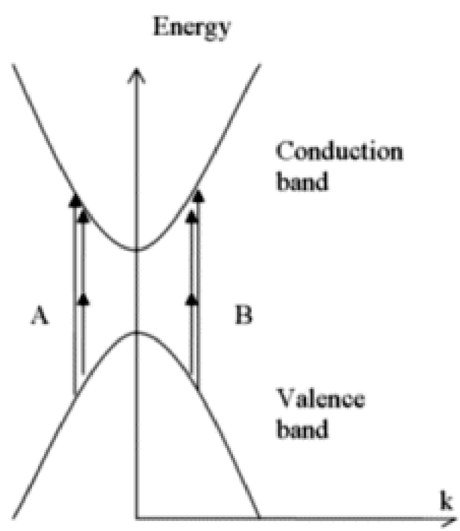
Experiment III. (Weinacht)

Controlling the phase in one arm of an optical interferometer relative to another allows one to control the amount of light that comes out. Analogously, coherent control uses the interference of two or more pathways between initial and final quantum states to control the total population transferred. Recently, researchers controlled the creation of photo-induced carriers ("photocurrent") in a semiconductor by simultaneously sending two linearly polarized laser pulses, one at frequency ω and the other at frequency 2ω, on the sample. (The laser pulses were weak enough for perturbation theory to be valid.) As is shown in the figure, light at 2ω is above the direct bandgap E_g of the semiconductor (i.e., $2\hbar\omega > E_g$), so absorption of one photon at 2ω can cause photocurrent. Light at ω is below the bandgap (i.e., $\hbar\omega < E_g$), so to create photocurrent with it requires "simultaneous" absorption of two photons of frequency ω. Absorption of one photon at 2ω and two photons at ω give two different paths to final states in the conduction band. These two paths can interfere. Controlling the relative phase of the two laser pulses allowed the researchers to control the direction of photocurrent in the semiconductor. For a given phase between the two lasers, the one- and two-photon paths constructively add to yield electrons propagating with negative momentum (A); for a phase 180^0 from this, electrons are promoted to the conduction band with a positive momentum (B).

a. (6 pts.) If you have a pulsed laser that provides light at frequency ω, how would you generate pulses of light at frequency 2ω? Draw a block diagram of an apparatus, label each block, and describe in a no more than a few sentences how each block works.

b. (6 pts.) Suppose the two laser pulses are not exactly harmonics: One laser has a frequency of ω and the other has a frequency of $2\omega + \delta$. If both pulses have a duration t, derive a simple expression for the minimum δ that causes the interference effect to go away. Be sure to explain in a few words why it goes away.

c. (8 pts.) Suppose, similar to above, one tries to use laser pulses at ω and 2ω to control the direction of photoionization current in a dilute gas of hydrogen atoms all initially in the ground state H(1s). Will this work? If yes, explain why; if no, explain why not. Assume that the peak electric field amplitudes of the 2ω pulse and the ω pulse are equal, not weak enough to be perturbative, and that the photon energy $\hbar\omega$ is one-twelfth of the ionization energy for H(1s). (Hint: The latter assumption means that the electromagnetic field varies slowly enough to be considered quasistatic. Therefore, the ionization mechanism is the same as occcurs in a static electric field. Consider what the total electric field amplitude looks like as a function of time as the relative phase between the ω-field and 2ω-field is varied.)



Experiment IV. (Gurvitch)

Describe various aspects and results of experiments that reveal the tunneling phenomena between normal metals and between superconductors. Include brief explanations of the relevant physics. While a detailed description would be much too long, you are expected to give a brief introduction to this field, as if trying to interest someone in taking a research project. The person you are trying to interest knows enough Physics, so that you can go beyond mere words, writing down relevant equations, sketching relevant graphs, etc.

a. (3 points) Start by briefly describing QM foundations of tunneling phenomena, which does not take place at all classically. You can discuss wave functions in the presence of a potential barrier, write Schroedinger's equation and discuss its solutions for different regions of space (particle incoming to a rectangular barrier, inside the barrier, outgoing particle); you can also discuss how this problem is solved; however, a complete solution is too involved and is not expected here. You can discuss the main properties of this solution. Use the Uncertainty Relation to get a rough estimate of the tunnel barrier thickness that will permit measurable tunneling (assume typical metals with $E_F = 10\text{eV}$ separated by an insulator with $\phi = 2\text{eV}$ barrier height).

b. (2 points) Sketch sample structure (metal layers, insulating layers, approximate thickness of different layers, cross-sectional and plan-view geometry of a tunnel junction), describe briefly how such samples can be made (you may include here some knowledge of relevant technologies, such as Nb-Al-Nb trilayer technology).

c. (2 points) Describe how such samples can be measured, which measuring instruments will you need? Sketch simple measuring circuits.

d. (3 points) Describe and explain what you expect to see if you study tunneling between normal metals at small voltages (mV range). Sketch an $I - V$ curve and comment on how this $I - V$ will change if you change (for example, increase) tunnel barrier thickness and change (for example, enlarge) tunneling sample area. (Hint: think of those electrons that can tunnel across the barrier when a modest voltage is applied across the barrier, and of the density of electronic states in normal metals on that energy scale). What if the voltage you apply is considerably higher, becoming comparable to the barrier height $\phi = (1-2)\text{V}$: what do you expect will happen to the tunneling conductance (slope $\mathrm{d}I/\mathrm{d}V$) of your junction's $I-V$ curve when V is that high? Explain with energy diagrams. What limits the maximum voltage that can be applied across the tunnel barrier?

e. (5 points) Describe what you expect to see if you study tunneling between two identical superconductors at $T << T_C$. In particular, describe the general shape of an $I - V$ curve at small (to tens of mV) voltages, and explain it based on what you know about superconductor's density of states. What can you deduce from such measurements? Use energy - density of states diagrams for both sides of the junction, similarly to part d, but for superconductors rather then for normal metals.

f. (5 points) Describe d.c. Josephson effect between two superconductors: what is observed and why? Describe (without a detailed explanation) the magnetic field dependence of the d.c. Josephson current considered to be a "signature" of the Josephson effect. Describe briefly the underlying physics (without magnetic field). Sketch a complete I-V curve containing features resulting from superconductor density of states and from dc Josephson effect.

Experiment V. (Wijers/Lattimer)

A star is observed in X rays with an integrated X-ray energy flux of $1.5 \cdot 10^{-10}$ erg cm^{-2} s^{-1}. The X-ray spectrum is consistent with a blackbody having a temperature of approximately 700,000 K. Further observations determine that the star has a proper motion of 300 milli-arcseconds per year and a parallax of 10 milli-arcseconds.

a. (7 points) From the above information, estimate the total luminosity of the star, its distance, and its radius. Explain and try to justify whatever assumptions you make. What kind of star is observed?

b. (5 points) Assuming that the source is a blackbody with the above temperature, show that its apparent V magnitude, m_V, is 25.6. The Planck function is

$$B_\lambda(T) = \frac{2\pi hc^2}{\lambda^5}[e^{hc/(\lambda kT)} - 1]^{-1} \text{ erg cm}^{-3} \text{ s}^{-1},$$

where $h = 6.63 \times 10^{-27}$ erg s, $c = 3 \times 10^{10}$ cm s^{-1}, and $k = 1.38 \times 10^{-16}$ erg K^{-1}, and the relation between specific flux at $\lambda = 550$ nm and V magnitude is

$$m_V = -2.5 \log_{10}\left(\frac{F_{\lambda=550 \text{ nm}}}{0.38 \text{ ergcm}^{-3} \text{ s}^{-1}}\right).$$

Give some reasons why you might expect this V magnitude estimate could be in error.

c. (8 points) An $m_V = 0$ source produces 1000 photons cm^{-2} nm^{-1} s^{-1} at $\lambda = 550$ nm. How long must one integrate through a V filter to detect this object at 5σ, if the telescope has a diameter of 8 m, the seeing is 1 arcsec, the sky background at V is 21 mag arcsec^{-2}, and the quantum efficiency of the total system (atmosphere + telescope + filter + CCD) is 40%. Approximate the image of the star as a uniformly illuminated square 1 arcsec on a side, neglect the variation of F_λ across the V band, and assume that the $V-$filter has a wavelength range of 100 nm with uniform transmission.

Experiment VI. (Peterson)

Establishing distances to distant objects in the Universe is a bootstrap process. Several techniques are involved, with one appropriate for nearer objects acting to calibrate the next one out. In some cases more than one technique operates over the same distance range, giving needed redundancy to the process.
The backbone of the process involves the Cepheid variable stars and the Period - Luminosity relation that they follow. The following questions touch on some of the important characteristics of this group of stars, the underlying physics and the accuracy of the distances so derived with current practice.

a. (2 points) Who discovered that the Cepheid variables might satisfy a linear relation between Period and Luminosity? What object containing these stars was being studied at the time?

b. (6 points) Describe the nature of the Cepheid variables. That is, are they very old? very young? massive? small? Where are they in their evolutionary path? Why would they pulsate?

c. (6 points) As a matter of practice, values from the theory of stellar evolution are not deemed secure enough, and the coefficients of the linear P-L relation are obtained observationally. Identify where the Cepheids are found that are used for this purpose. What will be the major sources of error in any calibration using these Cepheids. Give some sense of the current claimed uncertainties.

d. (6 points) In applying this relation to Cepheids in galaxies whose distances you wish to estimate, what are the major sources of error? How do you minimize them?

"Breadth"

Breadth I. (Sterman)

Electrons are produced in muon (μ^-) decay, in addition to neutrinos.

a. What neutrinos are produced?

b. If a muon decays at rest, what is the maximum and minimum energy of the produced electron (neglecting neutrino masses but not the electron mass)?

c. What is the mechanism for this decay in the standard model?

d. The angular distributions of electrons observed in muon decay depends on the quantity $\vec{p} \cdot \vec{s}$, where $\vec{p}$ is the momentum of the electron and $\vec{s}$ is the spin of the muon. Explain why this indicates that parity violation occurs in muon decay.

Breadth II. (Shuryak)

The most dense matter in Universe is at the center of the so called neutron stars, with the density n_0 about 1 baryon (3 quarks) per fm^3 ($1fm = 10^{-15}m = \frac{\hbar c}{198 MeV}$). It turns out that one can derive some of the basic properties of such stars using textbook statistical mechanics.

a. (8 points) Assume that the matter is an ideal Fermi gas of light up and down (u,d) quarks with negligible masses and at negligible temperature. Assume that there are no electrons in the system (model 1). Electric charges of u and d quarks are 2/3 and -1/3 respectively (electron charge is -1). Write down the condition of electroneutrality (the total electric charge of the star is zero). Calculate the Fermi momenta p_u and p_d of quarks in MeV/c and total energy density ρ_ϵ in MeV/fm^3 of the system. (Quarks have 2 spin orientations and 3 "colors".)

b. (8 points) There is an allowed weak reaction $d+\nu \leftrightarrow u+e$ which leads to to the production of electrons in the system. Write down the condition of chemical equilibrium between Fermi gases of quarks and electrons. Assume that neutrino ν leaves the star and can be ignored. Find the Fermi momentum of electrons p_e and Fermi momenta p_u, p_d, of quarks in MeV/c.

Hint: You can use the fact that p_e/p_u comes out to be a relatively small number and one can neglect higher powers of p_e/p_u.

c. (4 points) The following questions should be answered without any additional calculations. The third ("strange") quark has a mass $m_s \approx 150\, MeV/c^2$. What should be the condition for strange quarks to appear in the star? Are they present for the density of the star given above? The strange quark has a charge -1/3, like d and it can participate in similar reaction $s + \nu \leftrightarrow u + e$.

Breadth III. (Koch)

This problem is about diatomic molecules. Because it has six parts, your answers for each part should be kept brief. Some call for a small calculation.

When two atoms are far apart the force between them is negligible, and when they are so close together that they almost completely overlap, both their nuclei and their electrons repel strongly. Between these extremes the atoms can be bound by attractive force(s) into a diatomic molecule.

In this problem consider the following: the *homonuclear* diatomic molecule N_2 and the *heteronuclear* diatomic molecule CO.

Two N atoms, each with $Z = 7$ and $A = 14$, make N_2, which has 14 electrons and overall mass number 28. One C atom, with $Z = 6$ and $A = 12$, and one O atom, with $Z = 8$ and $A = 16$, make CO, which also has 14 electrons and overall mass number 28. One would expect N_2 and CO to have some similar properties, but there are important differences such as chemical reactivity.

In the problem, use the following information. The equilibrium separation between C and O in the lowest quantum state of CO is $R_e = 0.133$ nm, and the effective force constant between the atoms at $R = R_e$ is 1.86×10^3 N/m. Planck's constant is $h = 6.63 \times 10^{-34}$ Js. One atomic mass unit u corresponds to 1.661×10^{-27} kg and 931.5 MeV/c^2.

a. (2 pts.) Make a plot of the interaction potential energy $V(R)$ versus the internuclear separation R for a diatomic molecule such as N_2 and CO. Choose $V(\infty) = 0$ when $R = \infty$. Label the axes and use convenient units. Identify and briefly discuss "interesting" part(s) of the plot.

b. (3 pts.) Why does it make good physical sense to separate the energy level structures in N_2 and CO into (i) "electronic" levels; (ii) "vibrational" levels; (iii) "rotational" levels?

c. (2 pts.) Transitions between quantum states result in the emission of electromagnetic radiation. In a "multipole expansion" the most intense is usually that from electric dipole (E1) radiation. Explain why.

d. (5 pts.) Make a calculation that estimates to better than a factor of two accuracy the frequency splitting (in Hz) between the ground and first excited vibrational energy levels in CO. How does the splitting between adjacent vibrational states depend on the vibrational quantum number V as V increases? Are E1 transitions between these levels allowed in CO? If yes, say why; if no, say why not.

e. (3 pts.) Are E1 transitions between the vibrational levels allowed in N_2? If yes, say why; if no, say why not.

f. (5 pts.) Make a calculation that estimates to better than a factor of two accuracy the frequency splitting (in Hz) between the ground and first excited rotational energy levels in CO. How does the splitting between adjacent rotational states depend on the rotational quantum number R as R increases?

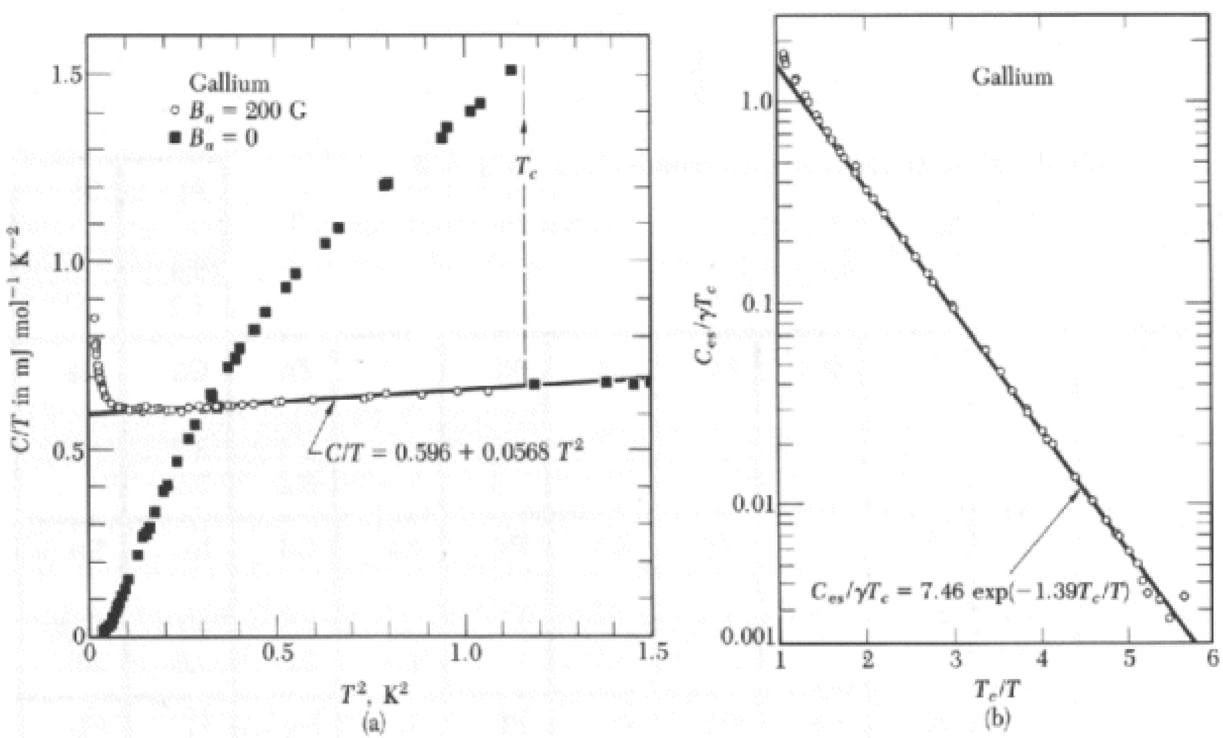
Breadth IV. (Mihaly)

The low temperature specific heat of Gallium is shown in the Figure. In the right panel the vertical axis is C/T in units of mJ/(mol K) and the horizontal axis is T^2 in units of K^2. The graph with the full squares was taken in zero external magnetic field. The data indicate a phase transition at $T_c = 1.07$K, where the specific heat has a sudden increase. In 200G magnetic field the data (empty circles) follow a straight line corresponding to $C/T = 0.596 + 0.0568T^2$. The upturn at very low temperature is due to nuclear quadrupole moments. In the left panel the electronic contribution is plotted in dimensionless units, with $C_{el}/\gamma T_c$ on the vertical axis and T/T_c on the horizontal axis. The straight line corresponds to $C_{el}/\gamma T_c = 7.46\exp(-1.39T_c/T)$, where $\gamma = 0.59$ mJ/(mol K^2)
(The Boltzman constant is $k_B = 1.38x10^{-23}$J/K$= 8.6 \cdot 10^{-5}$eV/K, Avogadro's number is $6 \cdot 10^{23}(1/\text{mol})$).

a. (4 points) What is kind of phase transition happens here and what is role of the magnetic field?

b. (4 points) Identify the source of the linear ($C \propto T$) and cubic ($C \propto T^3$) terms in the specific heat.

c. (12 points) Use the experimental results to calculate order of magnitude estimates for the Debye temperature and the Fermi temperature of Gallium. Calculate the magnitude of the energy gap in units of eV. (Note: you are not supposed to remember or derive the dimensionless quantities appearing in the equations.)



Breadth V. (Evans)

The Tully-Fisher Relation makes use of neutral hydrogen rotation curves to measure distances to nearby spiral galaxies.

a. (10 points) The stellar luminosity, L, of a spiral galaxy can be expressed

$$L \sim I_0 r_0^2,$$

where I_0 is the central surface brightness and r_0 is a scale radius. Derive the Tully-Fisher relationship between the stellar luminosity of a spiral galaxy and the velocity of the neutral hydrogen gas. For this derivation, make the assumption that the gas is in circular orbits. What other assumptions do you make, specifically, concerning the central surface brightness and the mass-to-light ratio of spiral galaxies?

b. (4 points) What can cause deviations from any of the assumptions used in A for $\lambda < 0.8\mu$m?

c. (6 points) How can this relation be used, for $\lambda > 0.8\mu$m, to measure distances?

Breadth VI. (Lattimer)

a. (10 points) Consider a solar-type star in which the opacity is of a Kramers type over much of the star. From dimensional arguments and the basic equations for stellar interiors, derive the mass–luminosity relation. Include the dependence on G, the gravitational constant. That is, determine the exponents α and β in the relation

$$L \propto G^\alpha M^\beta.$$

You may assume that the dominant nuclear energy source is the proton-proton cycle, and you may neglect the changing composition of the star as it ages.

b. (10 points) Consider a cosmological theory in which G varies inversely with time,

$$G = G_0(t_0/t),$$

where G_0 is the present value of G and t_0 is the present time. Deduce the luminosity of the solar-type star at time t in the past, and estimate the lifetime τ of the star. In the universe in which G is constant, assume the lifetime of the star is τ_0 and take the present age of the universe to be 15 billion years. From your results, what arguments would you make in favor of or against the theory that G varies inversely with time?