STONY BROOK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY

Tuesday, 3 September 2003 – Day 1

Qualifying Examination in Classical Mechanics and Special Relativity and in Electromagnetism and Optics

General instructions: Each area is graded separately. Do two of the three problems in one or both areas. Each problem is worth twenty points. If a problem has subparts, each of these will be equally weighted, unless indicated otherwise, with the sum totaling twenty points. Use one examination book per problem and label it carefully with your name, the name of the problem's author, and the date. You may not use any materials other than this examination paper and the exam books supplied, a calculator, your one page help sheet, and, with the proctor's approval, a foreign language dictionary. None of these materials may be shared between students.

Classical Mechanics and Special Relativity

Three problems, work any two.

CM I. (McGrew)

A beam of π^+ particles with momentum $p_{\pi^+} = 2$ GeV/c is prepared at an accelerator laboratory. The rest mass of a pion is $m_{\pi^+} = 140$ MeV/c^2.

a. (5 points) In the laboratory frame, how much time is required for a π^+ in the beam to travel a distance 10 meters?

b. (5 points) A π^+ at rest has a lifetime of 26 ns. What is the average distance that a π^+ in the beam will travel before it decays?

c. (10 points) A π^+ decays to a positive muon and a muon neutrino ($\pi^+ \to \mu^+\nu_\mu$) where the masses of the decay products are $m_{\mu^+} = 105$ MeV/c^2, and $m_{\nu_\mu} = 0$ MeV/c^2. What is the observed energy of the neutrino when it is found to have traveled at an angle of 0.25 rad to the beam direction? Use a Lorentz transformation from the center-of-mass frame to the laboratory frame, and give your answer in MeV.

CM II. (van Nieuwenhuizen)

A homogeneous ball of mass M and radius $r < R$ moves inside a fixed hollow sphere of radius R. The ball can only roll without slipping.

a. (4 points) Compute the inertial moment I of the ball.

b. (8 points) What is the minimum velocity the ball must have at the bottom of the sphere so that it can reach the top of the sphere?

c. (8 points) What is the frequency of the small oscillations of the ball around its equilibrium position at the bottom of the sphere along a meridian of the sphere (linear motion).

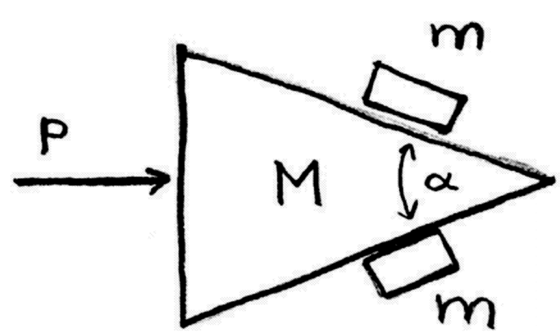
CM III. (Mihaly)

A wedge-shaped object, of central angle $\alpha = 30^o$ and mass $M = 1.0$ kg rests on a frictionless, horizontal plate. The plate also supports two blocks of equal mass $m = 0.2$ kg, that are in contact with the wedge in symmetric positions (see Figure). The coefficient of friction between the blocks and the wedge is μ. (Assume that the coefficients of static and kinetic friction are the same.) An impulse ("kick") of $p = 12.0$kg m/s is applied on the wedge along the symmetry axis.

a. (6 points) Make "free body diagrams", i.e. indicate on separate drawings the forces acting on the wedge and on the blocks. It is sufficient to draw one of the blocks, and use symmetry to treat the other. Make sure that the direction of the friction force between the wedge and the block is indicated. Discuss the constraint that connects the velocity (or acceleration) of the sliding block and the wedge. Indicate the reference frame(s) used to write the vector components.

b. (7 points) At least how large should be the friction coefficient μ if the three object will remain together after the impulse is applied?

c. (7 points) Assume there is no friction. What will be the direction and magnitude of the velocity of the wedge and the blocks?



Electricity and Magnetism and Optics

Three problems, work any two.

EM&O I. (van Nieuwenhuizen)

Consider a pointlike magnetic dipole $\vec{\mu}$ at the origin.

a. (5 points) What is the vector potential $\vec{A}$ for this system. Compute the magnetic field $\vec{B}$ outside the origin, using the expression for $\vec{A}$.

b. (5 points) There is also a term in $\vec{B}$ proportional to the delta function $\delta^3(\vec{r})$. Calculate this term using that averaging $\partial_i \partial_j \frac{1}{r}$ over all angles yields $\frac{1}{3}\delta_{ij}\nabla^2\frac{1}{r}$. (Hint: use the formula curl (curl $\vec{V}$) = grad (div $\vec{V}$) $-$ $\vec{\nabla}^2\vec{V}$).

c. (5 points) Now consider a uniformly magnetized sphere of radius R and magnetization (magnetic moment density) $\vec{M}$. Prove that the fields $\vec{B}$ and $\vec{H}$ are constant inside the sphere. Assuming that outside the sphere the magnetic field is the same as that of a pointlike magnetic dipole, compute the magnetic fields $\vec{B}$ and $\vec{H}$ inside the sphere.

d. (5 points) Finally take the limit R tending to zero, and reobtain the result in b).

EM&O II. (Weisberger)

A microwave antenna radiating at a frequency f = 12 GHz is to be protected from the environment by a non-magnetic plastic shield of dielectric constant 2.5. The shield is a big hemisphere so that we can assume the radiation is normally incident and the radius of the shield is much larger than the wavelength.

a. (10 points) What are the boundary conditions satisfied by the fields at the inside and outside of the shield?

b. (10 points) From the results of part a) determine the minimum thickness of the shielding which will allow perfect transmission (no reflected signal).

EM&O III. (Sprouse)

Two slits are illuminated with coherent light from a laser with wavelength 632 nm. Fig. 1 shows the intensity pattern observed on a screen 3 m from the slits.

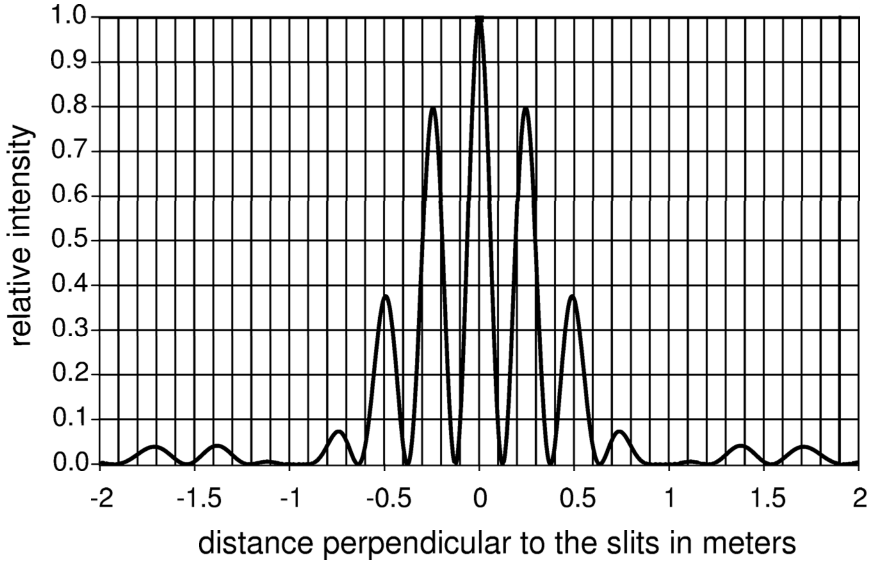


Figure 1: Interference from two slits illuminated by 632 nm light on a screen 3 m away from the slits.

a. (5 points) What is the spacing of the two slits from each other?

b. (5 points) What is the width of each slit?

c. (5 points) A piece of glass with index of refraction n=1.5 is placed over the two slits. Because the glass is not of uniform thickness, the glass is 100.00 μm thick over one slit and 100.632 μm thick over the other slit. What happens to the intensity of the pattern ant the position on the screen that is equidistant from the two silts (zero in the figure above)?

d. (5 points) The glass is removed and a linear polarizer is placed over one of the slits. A second linear polarizer with polarization axis perpendicular to the first one is placed over the second slit. The laser polarization is placed at 45°, bisecting the angle between the two polarizers. Sketch quantitatively the shape of the relative intensity pattern that will be seen on the screen.