

STONY BROOK UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

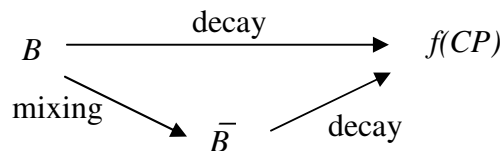
Comprehensive Examination, January 28, 2004

General instructions: Twelve problems are given. You should do any four, subject to the constraint that you must answer at least one from the "experiment" and one from "breadth" category. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used. You will find a list of useful constants at the end of the exam.

"Experiment"

Experiment I (Drees)

One of the outstanding question in fundamental science is why the Universe contains matter but very little antimatter. As discussed in a recent colloquium, the BaBar experiment at SLAC's e^+e^- collider PEP-II is looking for differences in the likelihood of certain decay products coming from a B compared to the likelihood of the anti particles of those products coming from a \bar{B} . The goal is to use these difference to decipher the tiny distinction between the laws of physics for antimatter and matter. Recently BaBar has observed the CP violation via the interference of mixed and unmixed B decays to a CP eigenstate $f(CP)$.



- (5 points) The "golden" B decay channel for BaBar is $B^0 \rightarrow J/\psi K_s^0$. Draw a Feynman diagram of the decay, given that the quark content of the B^0 is $\bar{b}d$. Why is this channel called the "golden" decay channel?
- (5 points) What is the Feynman diagram for $B^0 \bar{B}^0$ mixing?

- c) (5 points) In the experiment a 9 GeV electron beam collides with a lower energy positron beam. The center of mass energy is tuned to the $Y(4s)$ resonance which has a mass of 10.58 GeV. Calculate the energy of the positron beam.
- d) (5 points) Since the B meson mass is 5.279 GeV the $Y(4s)$ mostly decays to $\bar{B}B$. The decays of the B -mesons are observed in the experiment. Give a relation between the observed decay length and the time after which the B meson decays. If the mean lifetime of the B mesons is $\tau=1.56 \cdot 10^{-12}$ s, what is the typical decay observed in the experiment? Why were the beam energies chosen asymmetrically?

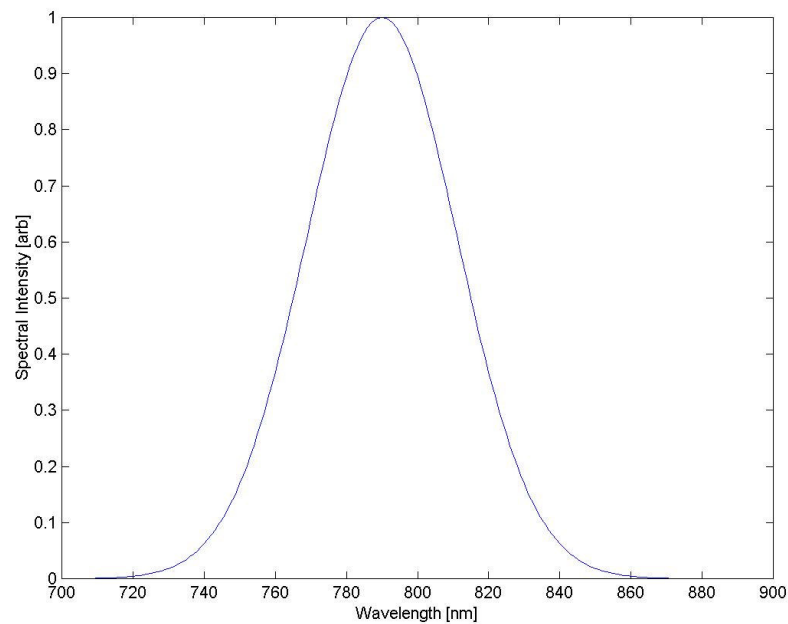
Experiment II (Sprouse)

Consider the following three questions.

- a) (7 points) For the nuclide ^{16}O the neutron and proton separation energies are 15.7 and 12.2 MeV respectively. Estimate the radius of this nucleus assuming that the particles are removed from its surface and that the difference in separation energies is due to the Coulomb potential energy of the proton.
- b) (7 points) In an experiment carried out with a beam of thermal neutrons it is found that on traversing a 2mm thick foil of ^{197}Au , some 70% of the neutrons are removed. What is the total thermal neutron cross-section for this isotope of gold? Comment on the result of the cross-section measurement in the light of the fact that the radius of a gold nucleus is 6.5×10^{-15} m.
- c) (6 points) A free neutron decays into a proton, electron and antineutrino. Assuming the latter to have negligible mass and the original neutron to be at rest, calculate the maximum momentum that could be carried off by the electron and compare this with the maximum momentum which the antineutrino could have.

Experiment III (Weinacht)

An optical spectrometer makes use of a grating to diffract light of different colors to different angles and therefore different positions in a detection plane some distance away. A Czerny Turner spectrometer uses a slit, two curved mirrors with focal lengths f , a diffraction grating and a spatially sensitive detector (such as a ccd camera or photodiode array).



- (4 points) Draw a block (schematic) diagram of what such a spectrometer might look like using the components listed. Explain your placement of these items and label distances between the components.
- (6 points) Why is there a slit and how does its size affect the spectral resolution of the spectrometer (for a fixed grating size and mirror focal length)? For a given spectrometer size and fixed mirror focal lengths, how does the dispersion (groove density) of the grating affect the spectral resolution (assuming arbitrarily high spatial resolution of the detector)?
- (10 points) Imagine you used the spectrometer you designed above to measure the spectrum of a short laser pulse shown below. Make a rough estimate of the shortest and longest pulse (FWHM of $I(t)$) that could have the frequency spectrum shown.

Experiment IV (Goldman)

A dilution refrigerator works by circulating ^3He through condensed mixture of the isotopes ^3He and ^4He . At temperatures below 0.8 K the mixture of the two isotopes separates into two distinct liquid phases: one floating on top of the other in a field of gravity (such as is the case on Earth). At the lowest temperatures one phase is nearly 100% pure ^3He ("concentrated" phase), and the other has about 6.4% of ^3He and the rest is ^4He ("dilute" phase). During operation ^3He is forced (by an external pump) to cross the concentrated-dilute phase boundary, a process somewhat analogous to evaporation, which causes cooling due to absorption of latent heat. The place where the phase boundary is maintained is called "mixing chamber".

- (4 points) Which phase floats on top, and why?

Imagine you have this mixing chamber maintained at a temperature $T_m = 10$ mK. Now, you want to vary the temperature of an "experiment" T_e , up to 200 mK. The experiment is

located in a chamber fully isolated from the outside world, except that it is connected to the mixing chamber by an $L = 30$ cm copper wire with resistance R . In order to maintain $T_e > T_m$, there is a resistive heater at T_e wound of the same copper wire with length $L_h = 120$ cm.

- b) (6 points) Making use of the Wiedemann-Franz law, derive the dependence of T_e on heater current I in terms of the parameters given above and the Lorentz number Λ . Use plausible simplifying assumptions and assume T_m is constant.
- c) (4 points) Sketch T_e versus I .
- d) (6 points) It is desirable to have the thermal link as strong as possible to minimize the thermal response time of the platform. If the maximum cooling power of the mixing chamber is $P = 0.1 \mu\text{W}$ at 10 mK, determine the minimum value of R that can be used if one wants to achieve $T_e = 200$ mK without heating the mixing chamber above 10 mK. Use the experimentally obtained value of $\Lambda = 2.5 \times 10^{-8} \text{ W Ohm/K}^2$ and neglect any other heat load on the mixing chamber.

Experiment V (Evans)

One way to study the large-scale inhomogeneity of the universe is to measure the velocities that it induces on observed objects, such as galaxies.

- a) (4 points) Explain why it is necessary to have a distance indicator other than the redshift in order to measure a peculiar velocity.
- b) (4 points) The fluxes of galaxies can, in principle, serve as distance indicators, but galaxies unfortunately have a large range of luminosities. (We say that they are poor “standard candles”.) The situation is remedied, in part, by taking advantage of phenomenological relations such as the Tully-Fisher relation between luminosity and velocity width for spiral galaxies.

$$L \propto \Delta v^4$$

Explain what causes the velocity width in spiral galaxies and why the Tully-Fisher relation is distance-independent.

- c) (4 points) Even with the use of the Tully-Fisher relation, flux-based distances are, at best, accurate to $\sim 15\%$. Consider the determination of the peculiar velocity of a galaxy whose Hubble expansion velocity is 4000 km/s. Show that the error in the measurement of its peculiar velocity is on the order of the peculiar velocity sought.

- d) (4 points) How can the study of large-scale motions in the universe be improved in view of the above observational limitation?
- e) (4 points) In practice, galaxies are selected from a flux-limited sample. Explain why this introduces a bias in distance measurements based on the Tully-Fisher relation. Show that the bias is such that the distances are, on average, overestimated more so the farther the galaxy.

Experiment VI (Lanzetta)

In this question we consider the determination of redshifts for quasars.

- a) (7 points) Show that the relativistic redshift for an object moving away from us at a velocity v ($c=1$) is given by:

$$\frac{\lambda_0}{\lambda} = \sqrt{\frac{1 - v}{1 + v}}$$

where λ_0 is the rest wavelength of the line and λ is the observed wavelength.

- b) (6 points) Quasars have a handful of well separated emission lines. For $v \ll c$, the redshift is small, and the lines can be identified because of their proximity to the rest wavelengths of known atomic transitions. This method fails for objects whose recession velocities are a significant fraction of c . How is line identification made in those cases?
- c) (6 points) In addition to their Hubble velocities due to the cosmological expansion, galaxies, and hence also the quasars in their nuclei, have so-called peculiar velocities on the order of a few hundred km/s. Show that the component of the redshift due to the peculiar velocity transverse to the line of sight is negligible compared with the component along the line of sight.

"Breadth"

Breadth I (McCarthy)

Consider $K^0 - \bar{K}^0$ mixing by making the simplifying approximation that CP symmetry is absolutely conserved in nature. K^0 (with valence quark content $\bar{s}d$) and \bar{K}^0 ($d\bar{s}$) are produced as flavor eigenstates by the strong interaction. However, they do not have definite mass and lifetime. Within the approximation of CP conservation, the states with definite mass and lifetime can be taken as the CP eigenstates with masses m_{\pm} and lifetimes τ_{\pm} (labeled by their CP eigenvalues ± 1).

- a) (6 points) Define the CP transformation (charge conjugation times parity) phases such that

$$CP | K^0 \rangle = | \bar{K}^0 \rangle$$

$$CP | \bar{K}^0 \rangle = | K^0 \rangle$$

and find two CP eigenstates of the neutral K -meson system.

- b) (7 points) Presume a K^0 is produced by the strong interaction at time $\tau = 0$ (in its rest frame). Find $P(t)$, the probability that the initial K^0 state is observed as a K^0 at time τ , in terms of m_{\pm} and τ_{\pm} .
- c) (7 points) The mass difference $\Delta m = m_- - m_+$ has been measured to be 5.35 ns^{-1} . Express this mass difference in MeV and use the known K^0 meson mass to estimate an experimental upper limit to the fractional mass difference between a particle and its antiparticle.

Breadth II (Shuryak)

The active ingredient of nuclear reactors, U^{235} has a half-life of $0.7 \cdot 10^9$ years and its natural fraction today is 0.7%. The common isotope U^{238} has half-life of $4.5 \cdot 10^9$ years. It was found that about $t_{oklo} = 1.8 \cdot 10^9$ years ago in the Oklo uranium mine (in Gabon, Africa) there was a naturally working nuclear reactor.

- a) (3 points) Based on this information, evaluate a fraction of U^{235} at that time. (Which was apparently sufficient for the chain reaction to be self-sustained.)

Fission of U^{235} creates many isotopes which do not occur naturally, so it is relatively easy to detect the existence of the reactor. The fission also produces a substantial flux of

neutrons n . Few isotopes, such as Sm^{149} , are known to have an unusually large cross section for n absorption, e.g. $\sigma_{149}^n \sim 70000$ barn, and as a result these isotopes are missing. This is indeed observed at the Oklo site, and it is estimated from the data that the capture cross section σ_{149}^n is within 20% the same as it is today. It turns out, this fact has remarkable consequences, providing about the best limits on the time stability of the fundamental constants of physics.

The cross section of neutron capture is large because in Sm^{149} there is a neutron resonance at very small energy $E_r = 0.0973 \pm 0.0002$ eV (which is large compared to its width).

- b) (7 points) From this information, and assuming that neutrons has thermal (near zero) energy, estimate what are the largest possible modification of the level position ΔE_r during this time t_{oklo} .
- c) (7 points) Considering Sm^{149} nuclei as a potential well for extra neutron with the fixed size and the depth $V \sim 50$ MeV, use the variation limit ΔE_r estimated above to establish the stability limits of the strength of nuclear forces $\Delta V/V$.
- d) (3 points) The same data are also used to get limits on time variation of the value of the electromagnetic charge e . Explain qualitatively how a change in electromagnetism affects the neutron levels.

Breadth III (Metcalf)

The first excited states of neutral 4He have one electron in the ground state with $n=1$ and the other electron in the next level with $n=2$. These are called the " $n=2$ states" even though one electron has $n=1$.

- a) (4 points) Enumerate the possible values of the quantum numbers for the various electronic angular momenta of these $n=2$ states.
- b) (4 points) Enumerate the possible quantum numbers of the projection of these angular momenta on an externally imposed z -axis, for example, by a magnetic field. If you think this is more easily done by a diagram, draw such a diagram.
- c) (4 points) Finding the wave function for these $n=2$ states as in part (1) above begins with a product of the wave function for each electron, say $|n_1, l_1, m_{l1}\rangle |s_1, m_{s1}\rangle |n_2, l_2, m_{l2}\rangle |s_2, m_{s2}\rangle$. However, this violates the Pauli principle even though the two electrons have different principal quantum numbers n_1 and n_2 . Explain why. What can be done to make a wave function that satisfies the Pauli principle?
- d) (4 points) Draw an energy level diagram of the states in part 1 above and also the ground state. Indicate the allowed and forbidden transitions between these $n=2$

states and the ground state in which both electrons have $n=1$, and provide an explanation of why some are forbidden.

- e) (4 points) Some but not all of these $n=2$ states have fine structure. Which ones, and why not the others? In the course of answering this, explain what is the origin of fine structure.

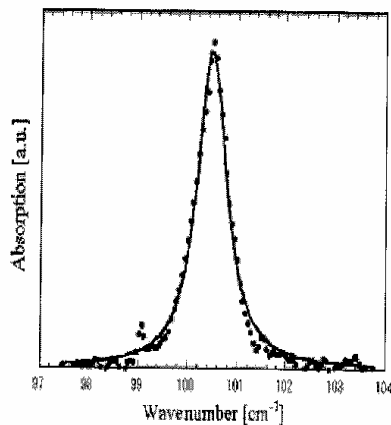
Breadth IV (Allen)

The data shown below for boron-doped Si and arsenic-doped Ge are experimental results taken from the literature showing the lowest energy absorption peaks for each. The concentrations of As and B are very low: much less than 1:1000.

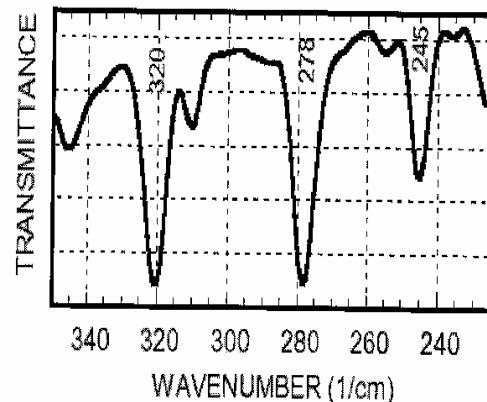
3B		4B		5B	
⁵	10.81	⁶	12.01	⁷	14.01
B		C		N	
(2030)	2.95	(3660)	2.2	-210	1.26
2650	2.0	4830	2.5	-196	3.0
¹³	26.98	¹⁴	28.09	¹⁵	30.97
Al		Si		P	
690	2.70	1410	2.39	44	1.82
2450	1.5	3690	1.8	290	2.1
³¹	69.72	³²	72.59	³³	74.91
Ga		Ge		As	
30	5.91	637	5.32	Subl.	5.72
2237	1.6	2830	1.8	.	2.0

- a) (5 points) Briefly describe what is being measured here and sketch what you would expect for similar measurements on pure Si and Ge. Explain any differences between the data presented and what you expect for pure Si and Ge.
- b) (5 points) Sketch a possible apparatus that could be used for these measurements. Describe its essential components and briefly discuss how to use it to take the data.
- c) (5 points) Explain the essential difference between boron and arsenic when used to dope semiconductors.
- d) (5 points) Use the data presented to calculate the ratio of the effective masses of carriers in the Ge conduction band to those in the Si valence band. You may use the information that the dielectric constant of Si is about 12 while for Ge it is about 16.

J. Kato, K. M. Itoh, E. E. Haller
arsenic-doped germanium

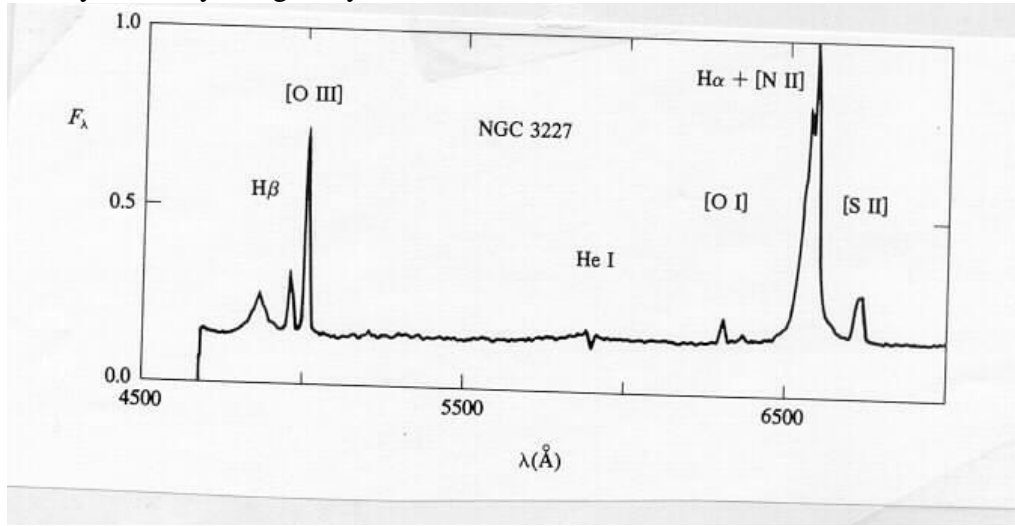


Daniel Krcho
boron-doped silicon



Breadth V (Simon)

Consider the spectrum of a Type 1 Seyfert galaxy given below. The flux per unit wavelength is normalized to the continuum, and is plotted versus wavelength in the rest frame of the galaxy. (However, at radial velocity ~ 1000 km/sec, the redshift is negligible for purposes of this question.) The species responsible for the spectral lines are identified. The lines of [SII] are actually a doublet but appear blended at the resolution of this spectrum. Similarly, the lines of $H\alpha$ and [NII] at 6563 and 6583 Å, respectively, are only marginally resolved.



- (8 points) The spectral lines in this spectrum provide important diagnostics about the physical conditions in the emitting region. With specific attention to the lines involved, describe as many of these diagnostics as you can.
- (6 points) The fractional abundances of oxygen and nitrogen relative to hydrogen are in the range 10^{-4} to 10^{-3} , yet the lines of these elements are detected at strengths comparable to those of hydrogen. Explain clearly, but without deriving the relevant formulae, how this situation arises.
- (6 points) Suppose the line emitting-region is angularly resolved, subtends a solid angle $\Delta\Omega$, and you have measured the total $H\alpha$ line flux. Describe the procedure to calculate the UV luminosity of the object exciting the region, and hence to estimate the number of O–B spectral type stars it contains.

Breadth VI (Lattimer)

In this question we consider the relationships between mass, luminosity and lifetime of a star.

- a) (5 points) What would be the characteristic timescale over which the radius of the Sun would change by a factor ~ 2 if it had no nuclear energy available to it?
- b) (5 points) Give two reasons why the scenario of part a. is untenable.
- c) (5 points) For stars whose masses are in the range 0.1–10 solar masses, the luminosity varies as

$$L \propto M^\alpha$$

With $\alpha=3-4$. Making the correct assumption that stars are fuelled by nuclear reactions in their cores, what can you deduce about the lifetimes of stars in this mass range?

- d) (5 points) Explain why the exponent of the mass–luminosity relation has to decline for massive stars, and asymptotically at high mass $\alpha \rightarrow 1$.

"Constants and Unit Conversions"

Avogadro number	$N_A = 6 \cdot 10^{23} \text{ mol}^{-1}$
Boltzmann constant	$k = 1.38 \cdot 10^{-23} \text{ J/K}$
Speed of light	$c = 3 \cdot 10^8 \text{ m/s}$
	$\hbar c = 197 \text{ MeV fm}$
electric charge	$e = 1.6 \cdot 10^{-19} \text{ C}$
fine structure constant	$\alpha = e^2/4\pi \epsilon_0 \hbar c = 1/137$
Gravitational constant	$G = 6.67 \cdot 10^{-11} \text{ m}^3/\text{kg s}^2$
Mass of Sun	$M_\odot = 1.99 \cdot 10^{30} \text{ kg}$
Radius of Sun	$R_\odot = 6.97 \cdot 10^8 \text{ m}$

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J}$$

$$1 \text{ eV}/c^2 = 1.78 \cdot 10^{-36} \text{ kg}$$

$$1 \text{ barn} = 1 \cdot 10^{-28} \text{ m}^2$$

Density of gold: 19300 kg m^{-3}

Particle masses:

$$m_{\text{proton}} = 938.27 \text{ MeV}/c^2$$

$$m_{\text{neutron}} = 939.57 \text{ MeV}/c^2$$

$$m_{K0} = 497.7 \text{ MeV}/c^2$$