STONY BROOK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY

Qualifying Examination **Classical Mechanics**, August 31, 2004

General instructions: Three problems are given. You should do any two. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used.

Classical Mechanics I (Likharev)

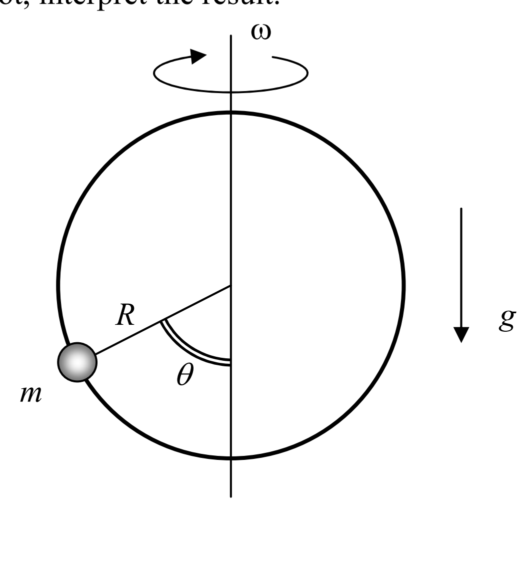
A bead may slide, without friction, on a circular hoop of radius R, that is rotated about its vertical diameter with a constant angular velocity ω – see Fig. below. Taking into account the uniform gravity field:

a) (6 points) Write down the kinetic energy K of the bead, its potential energy U, full energy $E = K + U$, and the Lagrangian $L = K - U$, as functions of angle θ of bead deviation from the lowest point.

b) (4 points) Find the Hamiltonian $H = \dot{\theta}\frac{\partial L}{\partial \dot{\theta}} - L$, where $\dot{\theta} = d\theta / dt$ is the generalized velocity corresponding to the coordinate θ of this system. Is H equal to E? Is H conserved in time? Is energy conserved? Comment on this situation.

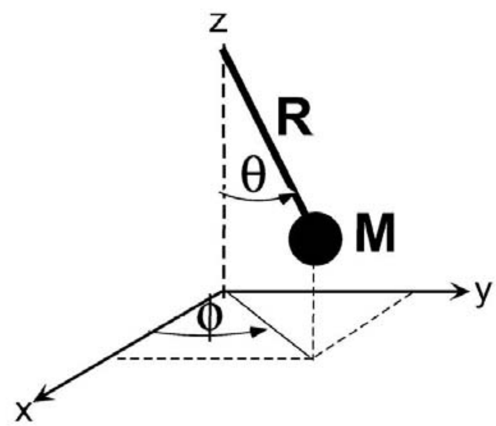
c) (6 points) Use the Lagrangian to write the explicit equation of motion for $\theta(t)$. Find the fixed point(s) θ_0 corresponding to bead equilibrium. Find the condition on ω for the existence of non-trivial fixed point(s) $\theta_0 \neq 0$.

d) (4 points) Find the frequency Ω of small oscillations about each fixed point. Are all Ω^2 always positive? If not, interpret the result.



Classical Mechanics II (Hobbs)

The figure shows a spherical pendulum consisting of a mass M connected to the bottom of a mass less rod of length R. The upper end of the rod is connected to a pivot which allows it to swing in any direction. Assume the system is in a constant gravitational field g.



a) (8 points) Using the Lagrange formalism, find the equations of motion for the pendulum in terms of the polar angle θ and the azimuthal angle ϕ (Do **not** assume small angles for the motion.

b) (5 points) Find all conserved quantities.

c) (5 points) Using any conserved quantities, eliminate as many coordinates from the problem as possible, and write an expression for the total energy, E. Using this expression, define an effective potential. Sketch the shape of this potential. How does it differ from a plane pendulum?

d) (2 points) Under appropriate conditions, a spherical pendulum can have stable circular orbits. Find a condition for these orbits in terms of the conserved quantities. (The result is a transcendental equation. Do not try to solve it.) Can one define the frequency of small oscillations about these circular orbits? If so, give a general expression for the frequency of such oscillations.

Classical Mechanics III (Verbaarschot)

Consider a particle in a Kepler-like central potential $V(r) = -G/r + \alpha/r^2$. Suppose that this potential describes the motion of a planet around a star (ideal situation).

a) (5 points) Which quantities are conserved for $\alpha = 0$? Draw the shape of the trajectories for negative and positive energy E.

b) (5 points) Argue that for $\alpha = 0$ the number of integrals of motion is exactly enough to get the solution for the trajectory of the planet by solving algebraic equations only, without integration. (You do not need to provide the explicit solution.)

c) (5 points) Which quantities are conserved for $\alpha \neq 0$? What values of the energy are allowed? Draw schematic pictures of the trajectory for small $\alpha > 0$ for negative energy E.

d) (5 points) Now consider the case when $\alpha < 0$. Draw the trajectories and derive the condition (on the energy E and impact parameter b) under which the planet falls onto the star. What is the trajectory separating falling and scattering trajectories?