

STONY BROOK UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

Comprehensive Examination, September 2 2004

General instructions: Twelve problems are given. You should do any four, subject to the constraint that you must answer at least one from the "experiment" and one from "breadth" category. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used. You will find a list of useful constants at the end of the exam.

"Experiment"

Experiment I (Averbeck)

The experiment BaBar studies CP violation in the B system at the e^+e^- collider PEP-II at the Stanford Linear Accelerator Center (SLAC). In this context, the measurement of B mesons decaying into charmless hadronic final states plays an important role. Recently, BaBar has reported the first observation of the extremely rare decay $B^0 \rightarrow \pi^0 \pi^0$.

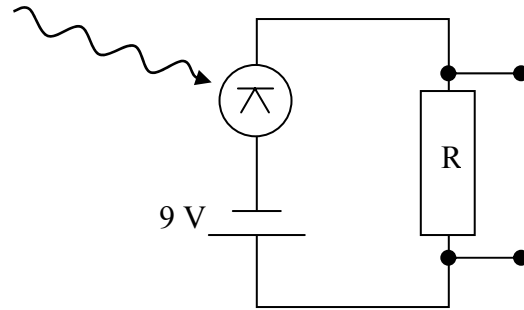
- a) (4 points) At SLAC, B mesons are produced in head-on collisions of 9.00 GeV electrons with 3.11 GeV positrons via the reaction $e^+e^- \rightarrow Y(4s) \rightarrow B\bar{B}$. The beam energies are chosen such that the intermediate $Y(4s)$ state is produced at rest in the center of mass system (CMS). Decreasing the energy of the positron beam to 2.49 GeV would result in the production of $Y(1s)$ states at rest. What are the masses of $Y(1s)$ and $Y(4s)$?
- b) (4 points) Draw a quark diagram for the decay $Y(4s) \rightarrow B^0 \bar{B}^0$. Why is this decay not possible for the $Y(1s)$ but possible for the $Y(4s)$ state? The quark content of the \bar{B}^0 is $b\bar{d}$ and its mass is 5.28 GeV.
- c) (4 points) Calculate the momenta of the \bar{B}^0 and the B^0 in the e^+e^- CMS. If in a particular event the B^0 is emitted in the direction of the positron beam, what are the momenta of the two B mesons in the laboratory system?

- d) (4 points) The two neutral pions from the subsequent rare decay $B^0 \rightarrow \pi^0 \pi^0$ are not detected directly but have to be reconstructed via their $\pi^0 \rightarrow \gamma\gamma$ decay. For that task BaBar uses a highly segmented electromagnetic calorimeter as many other experiments do. Describe briefly how neutral pions are reconstructed in such a detector.
- e) (4 points) BaBar has recorded 124 million $Y(4s) \rightarrow B\bar{B}$ events, with equal probability to produce charged or neutral $B\bar{B}$ pairs. In this data sample 46 ± 13 (stat.) ± 3 (sys.) $B^0(\bar{B}^0) \rightarrow \pi^0 \pi^0$ decays were reconstructed, where B^0 and \bar{B}^0 decays could not be distinguished from each other. The efficiency to detect a decay is given as 0.177 ± 0.027 (sys.) with negligible statistical uncertainties. Calculate the branching ratio (including statistical and systematic uncertainties) of this decay, i.e. the probability for B^0 or \bar{B}^0 to decay into two neutral pions.

Experiment II (Pietralla)

The heavy nucleus ^{160}Gd has a first and second excited states with spin J and parity π $J^\pi = 2_1^+$ and 4_1^+ at energies $E(2_1^+) = 75$ keV and $E(4_1^+) = 250$ keV.

- a) (6 points) Express the Hamiltonian for rotational motion of an axially symmetric quantum object in analogy to the classical expression for the kinetic energy of rotational motion. Give its eigenvalues in terms of the total angular momentum quantum number J . Why is the nucleus ^{160}Gd considered a rotational nucleus? Predict the excitation energy, spin and parity quantum number of the third excited state (with even spin quantum number!).
- b) (6 points) Suppose you perform a gamma-ray spectroscopy experiment on this nucleus. A radioactive ion beam facility provides a beam of $^{108}_{154}\text{Nd}$ ions per second on a ^9Be target with a thickness of 0.5 mg/cm². Determine the production rate of ^{160}Gd nuclei assuming that the $^9\text{Be} + ^{154}\text{Nd} \rightarrow 3n + ^{160}\text{Gd}$ reaction has a cross section of 1 barn. What is the count rate in your γ -ray detector if the detector had a detection efficiency of 5% and the average number of γ -quanta per reaction was 10?
- c) (8 points) Suppose you observe two close-lying states with identical quantum numbers $J^\pi = 4^+$ at excitation energies of 925 keV and 1075 keV, respectively. From the analysis of the corresponding rotational bands, you would have expected these two levels at unperturbed energies of 944 keV and 1056 keV, respectively. Determine the size of the mixing matrix element V between these two states that causes the energy shift! Employ a two-state mixing scenario.

Experiment III (Koch)

The above circuit shows a Si p-i-n photodiode in a series circuit with a 9 V battery and a resistor. The quantum efficiency of the Si photodiode at $\lambda = 800$ nm is $\eta = 0.8$, and its active area is 5 mm x 5 mm square. The wiggly line depicts incoming photons. The terminals on either end of the resistor R (specified below) indicate that the output signal is to be measured there with, e.g., an oscilloscope. Assume that the oscilloscope is a standard teaching-laboratory model with a bandwidth of 60 MHz and a maximum sensitivity of 5 mV per centimeter.

- (2 points) Is the Si diode forward biased or reverse biased? Does it make any difference in its application here as a photo detector?
- (2 points) Write a few sentences, with formulas as needed, that explain what is meant by “quantum efficiency η ”.

From a distance of 10 meters, a 1 mW laser operating continuous-wave (cw) at $\lambda = 800$ nm is directed at the Si photodiode detector. Both the laser and the detector are in a room with the lights on. Assume that the entire laser beam falls within the active area of the photodiode.

- (3 points) If the resistor in the circuit is $R=10$ k Ω , what signal will the oscilloscope register?
- (3 points) If the resistor in the circuit is changed to $R=100$ k Ω , what signal will oscilloscope register?

Now the 1 mW laser is removed and replaced by a spherical incandescent light bulb surrounded by a spherical shell of filter material that transmits EM radiation within a +/- 10 nm bandwidth centered at 800 nm. Assume that the total light bulb power transmitted by the spherical shell of filter material is 1 mW. The Si photodiode detector is still 10 meters away, and the lights in the room are still on.

- e) (5 points) If the resistor $R = 100 \text{ k}\Omega$, what signal will the oscilloscope register? If you are watching the screen of the oscilloscope and ask your assistant to block (put a hand in front of) and unblock (remove the hand from) the path between the light bulb and the Si photodiode detector, explain carefully why you will **not** be able to see any difference on the oscilloscope?
- f) (5 points) Still using the Si photodiode detector circuit with $R=100 \text{ k}\Omega$, describe carefully what other instruments you would need and use to be able to detect whether the light bulb is on or off in the presence of the ambient light in the room?

Experiment IV (Stephens)

A sample of germanium shows no Hall effect. If the mobility of the electrons in germanium is $3500 \text{ cm}^2/\text{V sec}$ and that of the holes is $1400 \text{ cm}^2/\text{V sec}$, what fraction of the current in the sample is carried by the electrons?

Experiment V (Simon)

About a hundred years ago Jeans showed that, in order for a cloud at density ρ and temperature T to collapse, it had to have a minimum size, or equivalently a minimum mass. While his derivation was technically incorrect, and is today known as the “Jeans swindle”, the corollary stands and is an important constraint on star formation in the interstellar medium.

- a) (5 points) Use dimensional analysis to estimate the free-fall time of a cold cloud with density ρ and size L .
- b) (5 points) With finite thermal (or turbulent) velocities, gravitational instability can be inhibited by sound waves which dissipate the growing density perturbation. Estimate the time needed for sound waves to dissipate density perturbations in the cloud as a function of temperature T , density ρ , and size L .
- c) (5 points) The Jeans criterion can be stated physically as “the faster process wins”. Use this criterion to estimate a minimum mass for the collapse of a perturbation at finite temperature under its own self gravity
- d) (5 points) Consider a typical cloud of neutral atomic hydrogen in the interstellar medium with a density of 100 cm^{-3} and a temperature of 100 K. Estimate the minimum mass it would need in order to collapse? Your estimate should be much larger than a typical stellar mass. What additional mechanism is at play in star formation?

Experiment VI (Lanzetta)

At red shift $z \sim 1000$, electrons combined with protons to form neutral hydrogen atoms. At some lower red shift, probably $6 < z < 30$, the Universe was reionized, presumably by ultraviolet-wavelength radiation from the first quasars. This question concerns a quasar embedded in the intergalactic medium prior to the reionization of the universe.

- a) (7 points) The quasar will ionize a region whose size depends on both the number of ionizing photons of the quasar and the density of the intergalactic medium. By equating the photo ionization and recombination rates, derive an expression for the radius of the “Strömngren sphere” ionized region surrounding the quasar in terms of the number of ionizing photons per unit time, Q , of the quasar, the hydrogen density n_H of the intergalactic medium, and the recombination coefficient α_B to excited levels of the hydrogen atom. Consider the intergalactic medium to contain only hydrogen and note that in highly ionized gas the electron density n_e and the proton density n_p are roughly the density n_H of the intergalactic medium.
- b) (6 points) Explain why the expression of part a. neglects recombination to the ground level of the hydrogen atom.
- c) (7 points) As the Universe expands, the density of the intergalactic medium drops, and the size of the Strömngren spheres surrounding quasars grows. Eventually, the spheres surrounding neighboring quasars overlap, and the reionization of the Universe is complete. Show that the red shift at which the Universe was reionized in terms of the *comoving* density Φ_0 of quasars and the *comoving* density n_{H0} of the intergalactic medium can be written as

$$z \approx \left(\frac{\Phi_0}{n_{H0}^2} \alpha_B \right)^{1/3} - 1 \quad .$$

Assume that all quasars emit the same specific number of ionizing photons per unit time, and note that *proper* densities exceed *comoving* densities by a factor $(1+z)^3$ due to the expansion of the Universe.

"Breadth"

Breadth I (Deshpande)

Most of what we know about the nucleon structure comes from lepton-nucleon deep inelastic scattering experiments. In this problem we will work out the consequences of electron-nucleon scattering.

- a) (4 points) Draw the Feynman diagram of e-p deep inelastic scattering.
- b) (3 points) What is the minimum electron energy needed to resolve structures within the nucleon? Assume the electron beam impinges on a proton target at rest.

The parton structure of the nucleon is expressed in terms of structure functions:

$$F_1^{eN}(x) = \sum_i^{N_q} f_i(x) Q_i^2 ; \quad F_2^{eN}(x) = x \sum_i^{N_q} f_i(x) Q_i^2$$

where Q_i is the charge of the parton (i) and $f_i(x)$ is the probability that the parton (i) carries a momentum fraction x of the proton.

- c) (4 points) Write down the structure function for protons and neutrons assuming a quark model with 2 flavors (u and d).
- d) (2 points) What is the ratio F_1^{ep} / F_1^{en} ? Assume $f_u(x) = f_d(x)$ and $f_{\bar{u}}(x) = f_{\bar{d}}(x) \sim 0$.

The fractional momentum carried by any particular parton flavor can be obtained by integrating over its momentum distribution.

$$P_u = \int_0^1 x(f_u(x) + f_{\bar{u}}(x))dx ; \quad P_d = \int_0^1 x(f_d(x) + f_{\bar{d}}(x))dx$$

Experimentally one can measure:

$$I_{ep} = \int_0^1 F_2^{ep}(x)dx = 0.18 ; \quad I_{en} = \int_0^1 F_2^{en}(x)dx = 0.12$$

- e) (3 points) Express I_{ep} and I_{en} in terms of P_u and P_d , assuming a quark model with two flavors. Solve for P_u and P_d .
- f) (4 points) Interpret the result.

Breadth II (Kuo)

- a) (5 points) Many large research laboratories are currently studying the so-called drip-line nuclei using a new type of accelerator. Describe briefly what are drip-line nuclei and what reactions can be used to reach the proton and the neutron drip-lines.
- b) (10 points) Oxygen ${}^{22}_8\text{O}$ is such a drip-line nucleus. Treating it as a collection of non-interacting fermions in a shell model, what is the shell model configuration for the ground state of this nucleus? Note that the shell model contains a spin-orbit potential of the form $f\vec{l} \cdot \vec{s}$ where the strength parameter f is negative.
- c) (5 points) Let J , T , and P denote the total angular momentum, total isospin, and total parity of a nucleus. Consider the nucleus ${}^{18}_9\text{F}$ as being composed of a closed ${}^{16}_8\text{O}$ core with valance nucleons confined in the $d_{5/2}$ orbit. Derive the allowed J , T , P values for ${}^{18}_9\text{F}$.

Breadth III (Weinacht)

In an optical lattice, cold atoms are trapped in an optical potential that can be considered harmonic near the minimum of the potential. Consider a Rb^{85} atom ($M_{\text{Rb}}=1.411 \cdot 10^{-25}$ kg) in the ground state of such a quasi harmonic potential ($V = \frac{1}{2} k x^2$, $k = 6.8 \cdot 10^{-15}$ J/m²) whose minimum is shifted from $x = 0$ to $x=a$. Describe the behavior of the atom if the potential is shifted by a small amount:

- a) (5 points) Suddenly
- b) (5 points) Adiabatically
- c) (5 points) Write down an expression for how slowly the potential would have to be shifted for the adiabatic approximation to be valid.
- d) (5 points) If the minimum of the potential is shifted very suddenly at $t = 0$ to $x = 50$ nm, how long will it take before the atom is at position $x = -50$ nm?

Breadth IV (Mendez)

Consider an electron in a one-dimensional periodic potential of period D such that the dispersion relation for the energy band of allowed states is described by

$$E(k) = \frac{\Delta(1 - \cos kD)}{2},$$

where k is the wave vector of the electron and Δ is a parameter that gives a measure of the width of the energy band.

- a) (1 point) Sketch the E vs. k relation for values of k within the first Brillouin zone. Make sure to label the axes, indicating the limiting values of E and k .
- b) (2 points) Calculate the group velocity of the electron as a function of k .
- c) (3 points) The electron, initially at rest and with total energy equal to zero, is accelerated by a uniform electric field F along the direction of the periodic potential. Write an expression for the dependence of k on time. Assume that F is small enough so that it is justified to use a semi classical approximation in which the band picture is not altered by the field, and that the electron does not suffer any collisions.
- d) (2 points) Calculate the group velocity of the electron as a function of time, when the field is present.
- e) (2 points) In that case, sketch the velocity of the electrons with time.
- f) (4 points) Use the equation of motion to show that under the external field F the electron undergoes an oscillatory motion of frequency eFD/h and amplitude $\Delta/2eF$.
- g) (4 points) Estimate the frequency and amplitude of those oscillations (*Bloch oscillations*) in a one-dimensional metal whose energy bandwidth Δ is 1eV, under a 1V/cm electric field. Estimate first the value of D for a metal such as aluminum.
- h) (2 points) Do you think it will be possible in practice to observe those oscillations in a solid? Why? If not, what assumption have we made here that may not be valid in a real solid?

Breadth V (Peterson/Lattimer)

The absorption lines seen in a simple stellar spectrum result from the absorption of radiation coming from deep in the atmosphere of the star by the cooler, outer layers of the atmosphere.

- a) (7 points) The fractional depth of an absorption line (anywhere in its profile), depends on the column density of absorbers along the line of sight. Show that, for weak absorption, the fractional depth of the absorption line is linearly proportional to the column density of the absorbers. Make sure to spell out the criterion for weak absorption.
- b) (6 points) In typical stars the Doppler effect dominates the broadening of the line cores. For an A0 star ($T=10000$ K), calculate the Doppler width of one of the D lines from the neutral sodium doublet ($\lambda=590$ nm), assuming that it is weak. Show that a Doppler absorption line has a Gaussian profile and compute the width between the two points on the profile at which the absorption is equal to $1/e$ of the peak absorption.
- c) (7 points) There is often insufficient spectral resolution to measure the line profile in detail, so one measures the fractional depth integrated over wavelength, known as the equivalent width. It should also be linearly proportional to the column density in the weak limit, and this can be used to determine if a line is weak. For example, in one star the equivalent widths of the D doublet lines of sodium (Na) were measured to be 0.024nm and 0.013nm, respectively, both ± 0.002 nm. Is this line weak? Why?

Breadth VI (Evans)

Groups and clusters of galaxies are the largest virialized structures from which one can determine the mass density of luminous matter and its associated dark matter. As such, the mass density in groups and clusters is of cosmological significance (cf., the colloquium by Spergel).

The luminosity function of galaxies in groups and clusters of galaxies can be approximated by the following relations:

$$n(L)dL = \phi^* L^{-1} dL \text{ for } L < L^*$$

$$n(L)dL = \phi^* \kappa L^{-7/3} dL \text{ for } L^* < L < 1000L^*$$

where $n(L)dL$ is the number of galaxies with luminosities between L and $L + dL$, $\phi^* = 0.001$ for a Hubble constant $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and L^* corresponds to an absolute B magnitude $M_B = -20.7$.

- a) (3 points) Solve for κ such that $n(L)$ is continuous at L^* .
- b) (3 points) If the absolute B magnitude of the Sun is $M_B = 5.4$, what is L^* in solar luminosities?
- c) (7 points) Assume that the mass-to-light ratio of galaxies varies with luminosity as $M/L = k L^{1/3}$, where k is a constant, and that $M/L = 80 M_{\text{sun}}/L_{\text{sun}}$ for an L^* galaxy. Use this and the luminosity function above to derive the equation for the total mass density in the Universe.
- d) (7 points) What is the mass density of the Universe, and how does it compare to the closure density of the universe ($= 5 \times 10^{-30} \text{ gm cm}^{-3}$)? Note that $1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$ and that $1 M_{\text{sun}} = 2 \times 10^{33} \text{ gm}$. Is this result consistent with the present understanding of the content of the universe?

"Constants and Unit Conversions"

Avogadro number	$N_A = 6 \cdot 10^{23} \text{ mol}^{-1}$
Boltzmann constant	$k = 1.38 \cdot 10^{-23} \text{ J/K}$
Planck's constant	$h = 6.62 \cdot 10^{-34} \text{ Js}$
Speed of light	$c = 3 \cdot 10^8 \text{ m/s}$
	$\hbar c = 197 \text{ MeV fm}$
electric charge	$e = 1.6 \cdot 10^{-19} \text{ C}$
fine structure constant	$\alpha = e^2/4\pi \epsilon_0 \hbar c = 1/137$
Gravitational constant	$G = 6.67 \cdot 10^{-11} \text{ m}^3/\text{kg s}^2$
Mass of Sun	$M_o = 1.99 \cdot 10^{30} \text{ kg}$
Radius of Sun	$R_o = 6.97 \cdot 10^8 \text{ m}$
Mass of the sodium atom:	$M_{\text{Na}} = 3.818 \times 10^{-26} \text{ kg}$
Sodium D ₁ oscillator strength:	$f_1 = 0.655$
Sodium D ₂ oscillator strength:	$f_2 = 0.327$
Atomic mass of Al:	27
Density of Al:	2.7 g/cm ³
1 eV	$= 1.6 \cdot 10^{-19} \text{ J}$
1 eV/c ²	$= 1.78 \cdot 10^{-36} \text{ kg}$
1 barn	$= 1 \cdot 10^{-28} \text{ m}^2$