STONY BROOK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY

Qualifying Examination **Electricity and Magnetism**, August 31, 2004

General instructions: Three problems are given. You should do any two. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used.

E&M I (Mihaly)

A slab of superconductor occupies the lower half-space, $z<0$. An infinitely long, thin wire currying current I is positioned parallel to the x axis at a height of $z=d$. For the purposes of this problem two properties of the superconductor are important: i./ the normal component of the magnetic field must be zero at its surface and ii./ there is no field inside the bulk of the material.

a) (5 points) What is the magnitude and direction of the force per unit length acting on the wire?

b) (5 points) Calculate the magnetic field for z>0. Formulate your result in terms of the two component of the field as a function of coordinates z and y: $B_y(z,y)$ and $B_z(z,y)$.

c) (5 points) Determine the direction of the surface current, and its dependence on the y coordinate, $j(y)$.

d) (5 points) Find the total surface current by integrating $j(y)$.

E&M II (Metcalf)

In a current carrying circuit with a long enough straight line section, we can consider a region close enough to the wire that an "infinitely long" approximation is suitable. Let the current be I and the radius of the wire be r_w.

a) (5 points) The symmetry of such a current element alone admits three symmetries of the magnetic field. Describe and sketch each one of them VERY carefully.

b) (5 points) Only one of these three is allowed by Maxwell's equations. Which one, and why are the other two forbidden?

c) (5 points) Suppose a uniform magnetic field $B = \mu_0 I / 5\, r_w$ is imposed perpendicular to such a current element. Show that there is a region where the field vanishes. How far is this region from the center of the wire? Draw the magnetic field lines in the plane perpendicular to the wire.

d) (5 points) Suppose there are two such elements closely spaced. Sketch carefully the magnetic field for the case that their currents are parallel and anti-parallel.

E&M III (Weisberger)

A spherical shell of radius a carries a uniform distribution of surface charge density with total charge Q. The shell rotates about the z axis with constant angular velocity $\vec{\omega}$.

a) (10 points) Find the magnetic dipole moment $\vec{m}$ of the sphere.

b) (10 points) The magnetic field (SI units are used) external to the sphere is

$$\vec{B}_e(\vec{r}) = \frac{\mu_0}{4\pi} \frac{3\hat{r}(\vec{m}\cdot\hat{r}) - \vec{m}}{r^3}$$

The interior field is

$$\vec{B}_i(\vec{r}) = \frac{\mu_0}{4\pi\, a^3}\vec{m}$$

Verify that these fields satisfy the appropriate boundary conditions at the surface of the sphere.