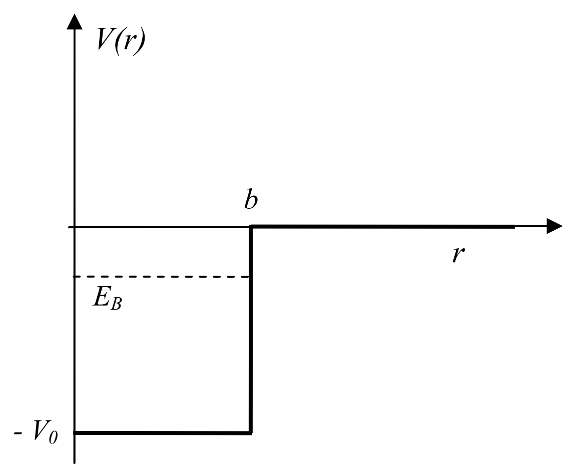
STONY BROOK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY

Qualifying Examination **Quantum Mechanics**, September 1, 2004

General instructions: Three problems are given. You should do any two. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used.

Quantum Mechanics I (Drees)

A deuteron is bound state of proton and neutron ($m_p \sim m_n \sim m \sim 939$ MeV/c^2). It can be modeled as one particle of reduced mass ($\mu = m_p\, m_n/(m_p+m_n)$) in a central square well potential of width b and depth $-V_0$, as shown in the figure. For the deuteron there is only one bound state with a binding energy of $E_B = -2.2$ MeV, which can be assumed to have no orbital angular momentum.



a) (6 points) Find the general solution for the radial wave function $u(r) = r\,R(r)$ for the two regions $r<b$ and $r>b$. (You do not need to normalize the wave function)

b) (6 points) Show that from the general solution you get a relation between E_B, V_0, b of the form:

$$\tan kb = -\frac{k}{k'}$$

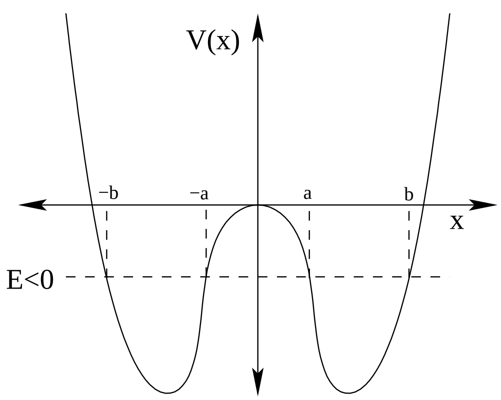
Where k, k' are functions of m, E_B and V_0.

c) (4 points) What is the width of the potential if its depths is $V_0 = 38.5$ MeV? (Use $\hbar c = 197\,fmMeV$ and note that kb must be positive!)

d) (4 points) Argue why there can not be a second bound state in this potential with $0>E>E_B$.

Quantum Mechanics II (Hemmick)

Shown in the figure below is a double well potential. We shall analyze this potential using the WKB approximation for the case of $E<0$ with 4 classical turning points labeled $\pm a$ and $\pm b$.



Please remember the following two WKB connection formulas:

$$\psi_-(x) \rightarrow 2\psi_S(x)$$
$$\psi_+(x) \rightarrow -\psi_C(x)$$

where

$$\psi_\pm(x) = \frac{1}{\sqrt{\rho(x)}}\exp\left(\pm\int \rho(x')dx'\right)$$

$$\psi_S(x) = \frac{1}{\sqrt{k(x)}}\sin\left(\int k(x')dx' + \frac{\pi}{4}\right)$$

$$\psi_C(x) = \frac{1}{\sqrt{k(x)}}\cos\left(\int k(x')dx' + \frac{\pi}{4}\right)$$

$$\rho(x) = \sqrt{\frac{2m(V(x)-E)}{\hbar^2}}$$
$$k(x) = \sqrt{\frac{2m(E-V(x))}{\hbar^2}}$$

a) (8 points) Write expressions for the wave function in the following three regions: ($x<-b$), ($-b<x<-a$), and ($-a<x<0$). You may have **only one** free normalization constant. (*Your solutions must match properly at the boundaries*).

b) (2 points) The potential is symmetric about the origin. This applies one of two restrictions to $\psi(x)$. State both conditions.

c) (7 points) Use the conditions from the previous step to prove the relation: $\tan(\phi(E)+\frac{\pi}{2}) = \pm\frac{1}{2\theta(E)}$

d) (3 points) Assuming $\theta >> 1$ (good regime for WKB), write down the condition between the quantum number n, $\phi(E_n)$, and $\theta(E_n)$.

To simplify your notation, please use the variables:

$$\phi(E) = \int_{-b}^{-a} k(x')dx' \, ; \; \theta(E) = \exp\int_{-a}^{a} \rho(x')dx'$$

<u>Quantum Mechanics III (Metcalf)</u>

Consider any two-level system such as an electron spin magnetic moment in a dc magnetic field or a two-level atom subject to laser light.

a) (2 points) Draw an energy level diagram of such a two-level system, labeling the states $|e\rangle$ and $|g\rangle$, having energies E_e and E_g respectively.

b) (8 points) The transition between these two states that can be driven by resonant electromagnetic radiation may be characterized by an off-diagonal matrix element of the Hamiltonian proportional to the radiation field strength, H_{eg}. Beginning with the Schrödinger equation, $H\psi = i\hbar\,\partial\psi/\partial t$, show why it is precisely H_{eg} that needs to be used.

c) (5 points) The fact that $H_{eg} \neq 0$ means that E_e and E_g are not the true energies of the states. Why not? Find the energies.

d) (5 points) The fact that $H_{eg} \neq 0$ means that $|e\rangle$ and $|g\rangle$ are not the true wave functions of the states. Find the wave functions.