

**STONY BROOK UNIVERSITY**  
**DEPARTMENT OF PHYSICS AND ASTRONOMY**

Qualifying Examination **Statistical Mechanics**, September 1, 2004

**General instructions:** Three problems are given. You should do any two. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used.

**Statistical Mechanics I (Zahed)**

A two-dimensional ideal, spinless and non-relativistic Bose gas is maintained in an area  $A$  with finite temperature  $T$  and chemical potential  $\mu$ , with  $z = e^{\mu/k_B T}$ .

- a) (8 points) Calculate the grand partition function  $\mathbf{Z}(z, A, T)$ . Separate the zero momentum part.
- b) (5 points) Calculate the average density of bosons  $\mathbf{n}(z, A, T)$ . Show that  $z$  must be less than 1 for any density.
- c) (2 points) Can the two-dimensional ideal gas Bose condense? Explain.
- d) (5 points) What changes in these arguments in three dimensions and why? (Hint: for nonzero momenta you may use the following expansion  $\ln(1/(1-x)) = \sum_{k=1}^{\infty} \frac{x^k}{k}$ , to do the integrals.)

**Statistical Mechanics II (Prakash)**

Consider a gas of non interacting particles with integer spins in equilibrium at temperature  $T$ . The thermal average number of particles in a state with energy  $\varepsilon$  is given by the Bose-Einstein distribution function

$$f(\varepsilon) \equiv \langle N(\varepsilon) \rangle = \frac{1}{e^{((\varepsilon-\mu)/\tau)} - 1}$$

where  $\tau = k_B T$  and  $\mu$  is the chemical potential.

- a) (8 points) Calculate the mean-square deviation  $\langle (\Delta N)^2 \rangle = \langle N^2 \rangle - \langle N \rangle^2 = \tau \partial \langle N \rangle / \partial \mu$  and the relative dispersion  $\langle (\Delta N)^2 \rangle / \langle N \rangle^2$ .

- b) (7 points) Under what physical conditions could a Bose-Einstein gas be thought of as a classical, i.e., Maxwell-Boltzmann, gas? What is the relative dispersion for a classical gas?
- c) (5 points) In which case, Bose-Einstein or Maxwell-Boltzmann, can the relative dispersion be made exceedingly small by increasing  $\langle N \rangle$ ?

### **Statistical Mechanics (Abanov)**

Consider a one-dimensional lattice with sites labeled by an integer number  $n=1, 2, \dots, 2N$ . The dimer configuration on this lattice is given by a set of bold links (dimers) connecting some nearest neighbors on the lattice. Dimers are hard core, i.e., there can be only one dimer originating at a given site of the lattice. Some typical configuration is shown in the Figure. Assume that dimers can be easily created or destroyed and that creating a dimer increases an energy of the system by  $\Delta > 0$ .



- a) (10 points) Calculate the free energy, entropy, and specific heat (per lattice site) of the system as functions of temperature, if the temperature is low  $T \ll |\Delta|$ .
- b) (3 points) Plot qualitatively specific heat vs. temperature for all temperatures.
- c) (7 points) How does a typical configuration look like for  $\Delta < 0$ ? Calculate the free energy, entropy, and specific heat for this case, still assuming low  $T \ll |\Delta|$ .