

STONY BROOK UNIVERSITY**DEPARTMENT OF PHYSICS AND ASTRONOMY**

Comprehensive Examination, January 26 2005

General instructions: Twelve problems are given. You should do any four, subject to the constraint that you must answer at least one from the "experiment" and one from "breadth" category. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used. You will find a list of useful constants at the end of the exam.

"Experiment"**Experiment I (Hobbs and van Nieuwenhuizen)**

Collider physics:

- a) (5 points) What are the beam particles at the Tevatron and at the LHC?
- b) (5 points) Write a Feynman diagram depicting the leading order production of a W boson at the Tevatron, and another diagram for the LHC. (Hint: one needs a quark-antiquark pair to produce a W boson)
- c) (5 points) Assuming the Higgs boson exists, the two dominant production modes of a Higgs boson at the Tevatron are a top-quark triangle coupled to two incoming gluons and an outgoing H, and the decay of a quark-antiquark into a virtual W (or Z) which subsequently decays into W + H (or Z + H). What are the experimental reasons for preferring the second mode? (Hint: think of backgrounds). At the LHC the preferred detection mode for a Higgs boson of mass less than 135 GeV is the rare decay $H \rightarrow \gamma\gamma$. Why?
- d) (5 points) How could one infer from $\gamma\gamma$ data at the LHC that the Higgs exists?

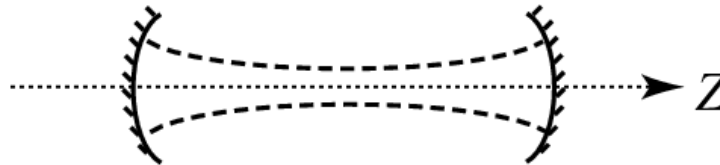
Experiment II (Drees)

The internal structure of nuclei can be described by a shell structure similar to that for electrons bound in atoms.

- (4 points) Describe the experimental evidence for the shell structure of protons and neutrons bound as nucleus.
- (6 points) Sketch the nuclear shell structure of ${}^4\text{He}$ (lowest lying s and p levels are sufficient) and compare it to that of atomic hydrogen. Discuss the main differences of nuclear and atomic shell structure.
- (6 points) What are the nuclear spin, parity, isospin and isospin projection of the ground state of ${}^4\text{He}$ and ${}^5\text{He}$, as well as of the first excited state of ${}^5\text{He}$? Explain your answer.
- (4 points) Explain how the spin of ${}^4\text{He}$ ground state and its first and second excited state can be measured. How can you deduce parity and isospin from experimental observations?

Experiment III (Metcalf)

One of the most common instruments to be found in any laser lab in atomic and molecular physics is the Fabry-Perot interferometer (FP). Its principle advantage is high precision for relative frequency measurements, but its principle disadvantage is ambiguity for absolute measurements.



- (4 points) How does this ambiguity arise? Explain the source of the ambiguity and describe its nature.
- (4 points) As shown in the diagram, FP's are usually made from curved mirrors in a confocal configuration because plane mirrors are hard to align and are only truly resonant for infinite size beams. What is the spacing L between mirrors in a confocal configuration whose radius of curvature is 15 cm?
- (4 points) If such mirrors have an intensity reflection coefficient of $r = 99\%$, give a numerical *estimate* the spectral resolution of such a FP (answer in MHz). Compare this resolution to the optical frequency.
- (8 points) The intensity of the optical field inside a FP illuminated by resonant light varies with position in all three coordinates, x ; y ; and z (z is the symmetry axis, see diagram). Describe this field in some detail, either by formulas and/or by drawings. Make a careful drawing of the wavefronts of the optical field.

Experiment IV (Mihaly)

Spin resonance is widely used in physics, chemistry and biology. The magnetization $\mathbf{M}(t)$ of the spin system is under the influence of the external field $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1(t)$ (where \mathbf{B}_0 is the static field and $\mathbf{B}_1(t)$ is the oscillating excitation field). Following the excitation by $\mathbf{B}_1(t)$, the spins precess around the static field. The instrument detects the magnetic dipole radiation emitted by the rotating spins (this signal is called the "free induction decay").

It is customary to write the equations in a rotating frame of reference. The z axis of this reference frame is lined up with the static field, and the angular velocity of the rotation equals to the excitation frequency ω . The response of the system is described by the quasi-classical Bloch equations for a general two-level system in an external field:

$$dM_z / dt = -\gamma M_y B_1 + (M_0 - M_z) / T_1$$

$$dM_x / dt = \gamma M_y b_0 - M_x / T_2$$

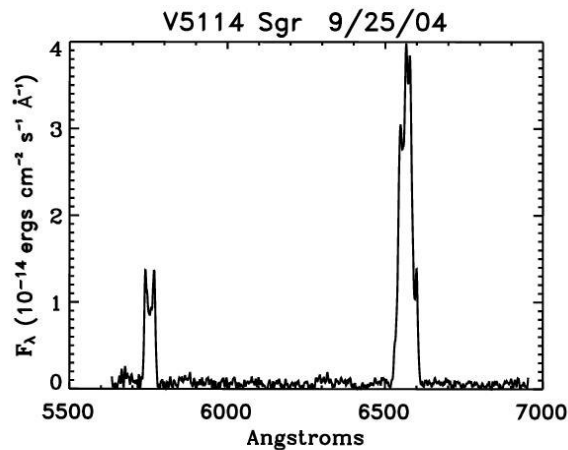
$$dM_y / dt = \gamma(M_z B_1 - M_x b_0) - M_y / T_2$$

Here the relaxation processes are characterized by two relaxation times, T_1 and T_2 , the M 's are components of the magnetization, M_0 is an equilibrium magnetization in a static field, and $b_0 = B_0 - \omega / \gamma$. The excitation field is represented by B_1 pointing in the x direction of the rotating frame of reference. The parameter γ is called the "gyromagnetic ratio"; for hydrogen nucleus $\gamma / 2\pi = 42.58$ MHz/T.

- (5 points) Assume $\mathbf{B}_1=0$. Give a simple physical reason for having two different relaxation times. What physical mechanisms are responsible for relaxation times T_1 and T_2 ?
- (5 points) Spin states are, in fact, quantized. In an infinitesimally small static field the population of "spin up" and "spin down" states are equal $n_{\text{up}}=n_{\text{down}}$. Estimate the ratio $(n_{\text{up}}-n_{\text{down}})/(n_{\text{up}}+n_{\text{down}})$ in a magnetic field of 0.5T for hydrogen nucleus at room temperature.
- (5 points) In pulsed spin resonance the excitation field is applied for a short time only, so that the magnetization of the sample is rotated by 90° , perpendicular to the static field. For an excitation field of $B_1=1 \times 10^{-3}$ T, how long the pulse should be? (Note: The static field is tuned to the resonance, $B_0 = \omega / \gamma$. You may assume that the pulse length is much shorter than T_1 or T_2 .)
- (5 points) In no more than one page describe the main components of a pulsed resonance spectrometer that can be used to measure NMR on hydrogen.

Experiment V (Evans)

- a) (7 points) An observer wishes to measure recession velocities of a sample of galaxies. The galaxies are assumed to have equal luminosities. What instrument should the observer use, and what galaxy ‘features’ will be observed to measure these recession velocities?
- b) (7 points) The observer wishes to make an estimate of the Hubble Constant by making observations of the same set of galaxies. What observations should the observer make, and why?
- c) (6 points) A few of the galaxies are observed to have negative recession velocities. What is a possible explanation for this?

Experiment VI (Walter)

The above picture is a portion of the spectrum of a nova about 5 months past outburst. The two lines are [N II] 5755 and H I 6563 (Balmer alpha).

- a) (10 points) Explain the shape of the [N II] 5755 line.
- b) (10 points) Explain why the H I line has a different shape.

In answering these questions, you should synthesize your knowledge about classical novae and atomic physics.

"Breadth"**Breadth I (van Nieuwenhuizen)**

- a) (3 points) Which particle/field exchanges lead to the 3 forces in the Standard Model? Draw for each force a simple Feynman diagram which exhibits this force.
- b) (4 points) Which are the quarks that make up a proton, a pion with positive charge, and a K meson with positive charge? Specify the $SU(3)\times SU(2)\times U(1)$ quantum numbers of the two lightest quarks.
- c) (5 points) What is the minimum energy an electron-antineutrino must have in order that it can produce a neutron and a positron when it hits a proton.
- In the center of mass frame.
 - In the frame where the proton is at rest. Give analytic formulas in terms of the masses, but no numerical result.
- d) (3 points) Write down the Feynman graph for the lowest-order radiative correction to the anomalous magnetic moment of the muon.
- e) (5 points)
- If one adds arbitrarily to the action for the Standard Model a mass term for the electron, instead of using the Higgs mechanism to produce such a mass term, does this violate any rigid (=global) and/or local symmetries? If your answer is negative, prove this; if it is affirmative, enumerate the symmetries that are broken.
 - What is approximately the value of the mass of the W boson?

Breadth II (Shuryak)

- a) (5 points) The gold nucleus has $Z=79$ protons, and the total number of nucleons in known isotopes is ranging from $A=Z+N=171$ to $A=205$. Using the simple model of a homogeneous non-interacting Fermi gas of protons and neutrons in a spherical well, calculate the Fermi momentum and kinetic energy for protons and neutrons. Use the values for the radius $R=6\text{ fm}$ and depth $U_0 \sim -57\text{ MeV}$.
- b) (7 points) Calculate the electromagnetic potential associated with its electric charge, assuming for simplicity that all charge is on the surface. Using this result and Fermi energies derived in part (a), conclude which isotope (in this model) could be the most stable isotope of gold? (The actual number is $N=118$) What kinds of decays are expected from unstable isotopes? (Ignore the proton-neutron mass difference.)
- c) (3 points) A neutron with negligible energy approaches a $N=118$ gold nucleus and is captured. What is the maximal amount of energy ΔE available for nuclear excitation?
- d) (5 points) The resulting excited nuclei come into thermal equilibrium and emit thermal photons, from which the magnitude of temperature T is deduced. It happens to be significantly larger than $\Delta E/(A+1)$: why? How does T depend on ΔE ?

Breadth III (Weinacht)

Consider exciting a vapor of Na atoms with steady light from the $3^2S_{1/2}$ ground state to the $3^2P_{3/2}$ state at $\lambda = 589\text{nm}$. The lifetime of the $3^2P_{3/2}$ state is $\tau = 16\text{ ns}$.

- a) (6 points) If you excite the atoms with light at the $3^2S - 3^2P_{3/2}$ resonance, what is the maximum steady-state population that you can achieve in the $3^2P_{3/2}$ state? Why?
- b) (7 Points) The Rabi frequency Ω characterizes the rate of excitation. What do you need to know to calculate Ω besides what's given above? How does Ω depend on light intensity?
- c) (6 Points) What is the general form of the equation for Ω if the optical frequency is detuned from the atomic resonance by an amount δ ? How is this related to the maximum achievable excited state population?

Breadth IV (McCoy)

Pick any element (such as hydrogen, helium, carbon, argon, iron, etc.) you wish and for this element:

- (10 points) Sketch its phase diagram in the P,T plane and give (approximately) the experimentally determined values of P and T for phase boundaries and critical points and give the nature of the order parameter in the various phases.
- (5 points) Describe how experimentally the phase diagram and order parameters of the phases are determined.
- (5 points) Give a theoretical microscopic explanation for the existence of the different phases on the phase diagram you choose in part a).

Breadth V (Yahil)

The nonrelativistic linear equation for small density perturbations in a pressureless, matter-dominated universe is

$$\frac{d^2\delta}{dt^2} + 2H(t)\frac{d\delta}{dt} - 4\pi G\rho_b(t)\delta = 0 . \quad (1)$$

Here $H(t) = R(t)^{-1} dR/dt$ is the Hubble function, $R(t)$ is the scale factor of the universe. $\delta = \rho/\rho_b - 1$ is the relative density perturbation away from the mean background density $\rho_b(t)$.

- (5 points) How would you interpret the second and third terms of Eq. (1)?
- (5 points) Write down the equation for the acceleration of the universe, d^2R/dt^2 , and use this equation to show that

$$\delta \propto H(t) \quad (2)$$

is a decaying solution of Eq. (1), i.e., show that $H(t)$ is a solution and that it decays with time.

- (5 points) Show that the other solution of Eq. (1) is:

$$\delta \propto D(t) = H(t) \int_0^t \left[\frac{dR(t')}{dt'} \right]^{-2} dt' \quad (3)$$

[Hint: look for a solution of the form $\delta = Hy$ and use the fact that H is a solution of Eq. (1).]

- (5 points) Neglecting the cosmological constant, how does the solution in Eq. (3) behave for cosmological density parameters $\Omega = 1$ and $\Omega = 0$? Deduce the past growth of perturbations in a universe whose current density parameter is $\Omega_0 = 0.25$ and the growth expected in the future. You may neglect dark energy, which has little effect on the growth of perturbations.

Breadth VI (Evans)

Stars form in molecular clouds comprised mostly of molecular hydrogen, but containing trace amounts of other molecules such as CO and HCN. The kinematics and quantity of molecular hydrogen can be traced by using the 115 GHz rotational transition of CO, which is collisionally excited by molecular hydrogen. In addition, the CO transition is observed to be optically thick to the CO emission in molecular clouds.

The CO luminosity, L'_{CO} of a spherical molecular cloud is given by,

$$L'_{CO} = \pi R^2 T_{CO} \Delta v \quad 1$$

where R is the radius of the cloud and $T_{CO} \Delta v$ is the flux of the CO emission line expressed in terms of the average radiation temperature over the area πR^2 and the velocity width of the line. For a cloud in virial equilibrium, twice the time-averaged kinematic energy is equal to the absolute value of the time-averaged potential energy.

- a) (8 points) Use equation (1) and the condition of virial equilibrium to derive an expression for the molecular hydrogen mass in terms of CO luminosity, the radiation temperature, and the cloud density. For this derivation, adopt the following expression for the kinetic energy:

$$0.5 M_{gas} \Delta v^2 \quad 2$$

where M_{gas} is the mass of molecular gas (i.e., the cloud mass) and Δv is the same velocity as in equation (1).

- b) (4 points) What must be true of the radiation temperature and density in order for the CO luminosity to be proportional to the molecular hydrogen mass? Is this physically feasible?
- c) (8 points) If CO observations of an ensemble of spatially resolved giant molecular clouds with a range of CO luminosities are obtained, how can these observations be used to test the proportionality between the CO luminosity and the molecular hydrogen mass?

"Constants and Unit Conversions"

Avogadro number	$N_A = 6 \times 10^{23} \text{ mol}^{-1}$
Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J/K}$
Planck's constant	$h = 6.62 \times 10^{-34} \text{ Js}$
Speed of light	$c = 3 \times 10^8 \text{ m/s}$
	$\hbar c = 197 \text{ MeV fm}$
electric charge	$e = 1.6 \times 10^{-19} \text{ C}$
fine structure constant	$\alpha = e^2/4\pi \epsilon_0 \hbar c = 1/137$
Gravitational constant	$G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2$
Mass of Sun	$M_o = 1.99 \times 10^{30} \text{ kg}$
Radius of Sun	$R_o = 6.97 \times 10^8 \text{ m}$

Mass of the sodium atom:	$M_{\text{Na}} = 3.818 \times 10^{-26} \text{ kg}$
Sodium D ₁ oscillator strength:	$f_1 = 0.655$
Sodium D ₂ oscillator strength:	$f_2 = 0.327$

Atomic mass of Al:	27
Density of Al:	2.7 g/cm^3

1 eV	$= 1.6 \times 10^{-19} \text{ J}$
1 eV/c ²	$= 1.78 \times 10^{-36} \text{ kg}$
1 barn	$= 1 \times 10^{-28} \text{ m}^2$