STONY BROOK UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

Comprehensive Examination, September 1, 2005

General instructions: Twelve problems are given. You should do any four, subject to the constraint that you must answer at least one from the "experiment" and one from "breadth" category. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used. You will find a list of useful constants at the end of the exam.

"Experiment"

Experiment I (Jung)

Neutrino physics:

a) (6 points) Does the sun (mainly) emit neutrinos or antineutrinos, and which is the dominant emission mode? Which neutrinos or antineutrinos are mainly produced in the atmosphere of the Earth, and by which process?

From the mixing equation for two neutrino species one may derive the survival probability of a given neutrino species as a function of the neutrino energy E and the distance L traveled by the neutrinos. Several solar neutrino experiments, using different L and E, have yielded a best fit for the mixing angle θ and the m^2 difference Δm^2, namely

$$\theta \sim 35^0 \text{ and } \sim 8\ 10^{-5}\ (\text{eV})^2$$

One of the crucial experiments that contribute to these results is the SNO (Sudburg Neutrino Observatory in Canada) experiment, which uses heavy water.

b) (7 points) Write down the 3 reactions of solar neutrinos in the SNO detector, due to elastic scattering off electrons, charged currents, and neutral currents, respectively. What are the gauge bosons whose exchange mediates the interactions in each of these reactions?
c) (7 points) Which of these 3 reactions also occur at Super-Kamiokande where one uses pure water? Explain how one can conclude at SNO that neutrino-flavor changing occurs by using the data of 2 of the 3 reactions.

Experiment II (Drees)

Jets are ideal tools to study the Quark Gluon Plasma created in central Au-Au collisions (impact parameter zero) at Brookhaven National Laboratory's relativistic heavy ion collider RHIC. Jets result from interactions between partons (quarks and/or gluons) involving a large momentum transfer. The scattered partons fragment into several hadrons within a ~30 degree cone around the parton direction. The spray of particles from a fragmenting parton is referred to as a jet.

a) (2 points) How does the four momentum of the "jet" relate to the four momentum of the struck parton?

b) (10 points) Consider a collision of two partons in a p-p collision at a center of mass energy of 200 GeV. Assume a jet with 20 GeV energy is emitted in the y direction perpendicular (at 90 degrees) to the beam axis (z direction). Assume one parton carries a fraction x_1=0.2 of the beam momentum, while the other one carries a fraction x_2=0.3. Calculate the momentum and energy of the second jet as observed in the laboratory.

c) (3 points) In a Au-Au collision several thousand particles are emitted nearly isotropically with thermal momentum distribution, reflecting a temperature of say 400 MeV. Propose a method to detect jets in this environment. (Hint: the most energetic hadron in a jet typically carries 30% of the jet's total energy.)

d) (3 points) One of the discoveries at RHIC was that jets are effectively absorbed in the opaque quark gluon plasma; let's say with an absorption length of 0.5 fm. Propose a measurement to observe the jet absorption.

e) (2 points) In Au-Au collisions at RHIC jets are still observed. How could they escape being absorbed?

Experiment III (Weinacht)

a) (8 points) What kind of device can you use to detect single optical photons? How does this device work? Draw a simple diagram of such a device and give a quick explanation of the basic principles involved.

b) (4 points) For each photon that is incident on your detector, what is the probability that you will detect it? What does this probability depend on?

c) (8 points) Give a back of the envelope calculation for what kind of signal (peak height, polarity and duration) you would generate from this device for detection of a single photon if you connected it to a 50 Ω channel on an oscilloscope. Be explicit about the assumptions you make.

Experiment IV (Abanov)

Gerd Binnig and Heinrich Rohrer were awarded the Nobel Prize of 1986 in physics "for their design of the scanning tunneling microscope". A schematic diagram of the Scanning Tunneling Microscope (STM) is shown in the Figure. The STM is the instrument which analyzes the surface topography of a conducting material by moving a miniature probe that has a single atom at its apex over the surface of the sample. The tunneling current between a metallic tip and a conducting substrate constant is maintained constant as the tip is scanned across the surface. The image is formed by plotting the tip height versus the lateral tip position.

a) (5 points) Briefly describe the main challenges in designing an STM.

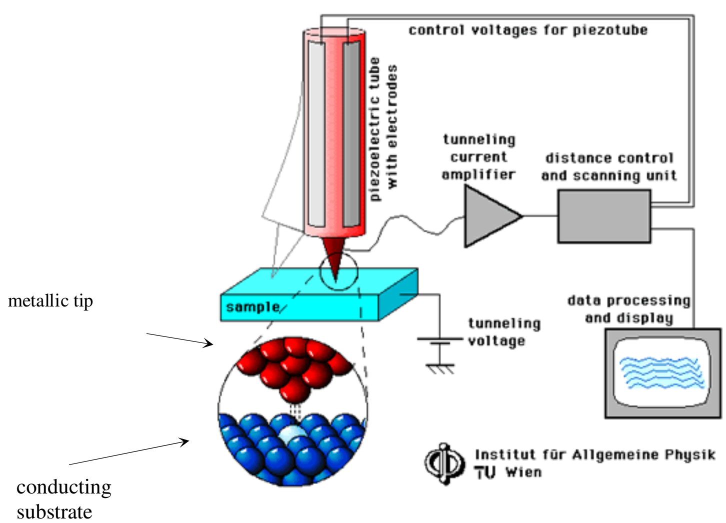
b) (5 points) Briefly describe the function of each element in the STM diagram (Not more than 1-2 lines per element).

c) (5 points) At low voltages and temperature the tunneling current is given by

$$I \propto \exp(-2Kd), \qquad (1)$$

where d is the distance between the tip and sample. Assuming that the local barrier height is about 4 eV, find the inverse decay length K. If the current is kept constant to within 2%, what is the instrument's sensitivity to the distance between the tip and the sample?

d) (5 points) The current depends exponentially on both gap distance and the local barrier height (see Eq. (1)). The change of the current may be due to both corrugation of the surface and to the locally varying local barrier height. Explain how these two effects can be separated. (*Hint:* The tip in STM is vibrating vertically with the small amplitude.)



Experiment V (Evans)

Comets are classified as either short-period or long-period comets. The short-period comets have fairly low orbital inclinations relative to the plane of the ecliptic, and aphelion distances (i.e., farthest distance from the Sun) of ~ 50 astronomical units, whereas long-period comets collectively have a random set of inclinations and aphelion distances of 50,000 astronomical units. Although comets are ultimately destroyed via sublimation and impacts with the Sun and planets, there is a steady stream of comets visiting the inner solar system.

a) (5 points) Explain the mechanism by which short and long-period comets are replenished.

b) (5 points) Devise an experiment to find the reservoir (i.e., the Kuiper Belt) from which short period comets emanate. Be specific about the type of instruments you would use, the part of the sky you would observe, and the manner in which you would identify such `Kuiper Belt Objects'.

c) (5 points) The albedo is a measure of the reflectivity of an astronomical object. How would you go about measuring the albedo of a Kuiper Belt Object? You should state the instruments you would use to obtain this measurement. State any assumptions you make.

d) (5 points) Finally, given the above set of observations, how would you estimate the size of a Kuiper Belt Object? State any assumptions you make.

Experiment VI (Walter)

The neutron star RX J185635-3754 has a spectrum that is very close to a black body at X-ray wavelengths. The blackbody temperature kT is 62 eV.

a) (5 points)What is the wavelength where the flux peaks? Recall that λ_{max} T = 3 10^7 A K for a black body. If you were designing a program to discover more stars like this, would you use a small X-ray telescope (ROSAT, ~ 50 cm effective diameter), the Hubble Space Telescope (2.4m aperture; UV/optical sensitivity), or the 8m diameter Gemini optical-near-IR telescopes? Explain your choice.

b) (5 points) The parallax is 0.008 seconds of arc (1 second is 1/3600 of a degree). What is the distance? You may use any units you like.
RX J185635-3754 is one of the closest known neutron stars. If there are 10^9 neutron stars in the galaxy, about how close do you expect the nearest one to be? Clearly state all assumptions you make about the volume of the galaxy. Compare your expectation to the distance, and comment.

c) (5 points) The luminosity of the neutron star is 1.5 x 10^{31} erg/s. What is the stellar radius? Is this reasonable for a neutron star? If the mass is 1.5 solar masses, how does the mean density compare to that of nuclear matter?

d) (5 points) With a 62 eV temperature, the age of RX J185835-3754 is about one million years. It seems to be moving away from the Upper Scorpius OB association, which has an age of about 5 million years. Is it reasonable that an OB association can have spawned a neutron star in this amount of time? Be quantitative. Estimate the lifetime of a 50 solar mass star, given that its main sequence luminosity is about 10^5 solar luminosities. One solar luminosity is 4×10^{33} erg/sec.

"Breadth"

Breadth I (van Nieuwenhuizen)

a) (5 points) Write down the electromagnetic covariant derivative for a complex scalar field ϕ. (The minimal coupling of a complex scalar field to electromagnetism). Show that taking constant ϕ leads to a mass of the photon. (The Higgs effect for photons).

b) (5 points) Write down the covariant derivative of a covariant vector field v_ρ in general relativity. Derive from this result the Riemann curvature tensor. (Hint: take a second covariant derivative).

c) (5 points) Show that the gauge field $A_1^a = x^0 F_{01}^a, A_0^a = A_2^a = A_3^a = 0$ satisfies the Yang-Mills field equations without matter for constant but arbitrary F_{01}^a.

d) (5 points) Write down all tree graphs (Feynman diagrams without loops) in the Standard Model for processes $a+b \rightarrow c+d$ for the scattering of an electron and an up-quark ($e^-+u \rightarrow \ldots + \ldots$). Identify all particles in each diagram.

Breadth II (Pietralla)

Neutrons and Protons represent the most important degrees of freedom for low-energy nuclear structures although they are not elementary particles.

a) (2 points) Give the quark content of protons and neutrons in the static quark model.

The nucleon-nucleon interaction at low energy cannot be calculated from QCD. The interaction of nucleons with the bulk of a heavy nucleus is often modeled in a phenomenological way.

b) (2 points) Give three qualitative properties of the nuclear force.

A useful approximation to the nuclear potential acting on a nucleon in a heavy nucleus is the harmonic oscillator. Including centrifugal and spin-orbit corrections the nucleon's single-particle energies can then be written as

$$E(N,j) = \hbar\varpi\left(N + \frac{3}{2}\right) + V_l \frac{\vec{l}\cdot\vec{l}}{\hbar^2} / + V_{ls}\frac{\vec{l}\cdot\vec{s}}{\hbar^2}$$

with angular momentum $\vec{l}$ the spin $\vec{s}$ operators and the phenomenological constants $\hbar\varpi$=40 MeV/A$^{1/3}$, V_l= −4 MeV/A$^{2/3}$, and V_{ls}= −20 MeV/A$^{2/3}$.

c) (4 points) Express $\frac{\vec{l}\cdot\vec{l}}{\hbar^2}$ and $\frac{\vec{l}\cdot\vec{s}}{\hbar^2}$ in terms of the orbital quantum number l for the two possible spin couplings $j = l \pm 1/2$.

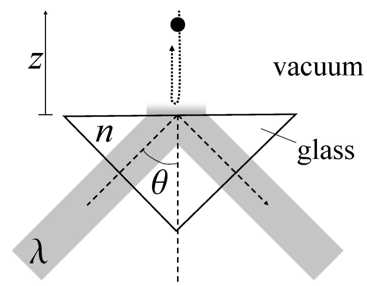
d) (4 points) Construct and draw the single-particle term scheme in relative units ($e = E/\hbar\varpi - 3/2$) for a nucleus with mass number A = 125 for all single particle states with N = 0 to 4. Identify the shell closures and give the "magic numbers" of particles at the shell closures.

e) (4 points) Use this scheme of single-particle levels for predicting the ground state angular momentum quantum numbers for odd-mass calcium isotopes (from A=37 to 51, Z=20).

f) (2 points) What are the total angular momentum quantum numbers J two protons in the $\pi(1f_{7/2})$ shell can couple to? Give a proof !

Breadth III (Schneble)

As illustrated in the figure, a glass prism surface covered with an evanescent light field can be used as a mirror for atoms. The evanescent field is formed by total internal reflection of a laser beam and gives rise to an optical potential $U(z) = U_0 \exp(-2z/\xi)$ above the glass surface, where z is the distance from the surface and ξ is the decay length of the evanescent field. If the prism surface is oriented horizontally (i.e. perpendicular to gravity g), an atom can bounce off the surface many times. With additional confinement parallel to the surface, one can create a 3D atom trap that can be used to study ultracold, two-dimensional atomic gases.



a) (5 points) Calculate the decay length ξ of the evanescent field amplitude in terms of the vacuum wavelength λ of the laser light, the refractive index n of the glass prism, and the angle of incidence θ.
b) (6 points) Calculate the location of the potential minimum normal to the surface. What is the extension l_{HO} of the ground state in the harmonic approximation? At what temperature T^* would you expect a freezing out of an atom's motional degree of freedom perpendicular to the surface? Compute l_{HO} and T^* for ^{133}Cs atoms trapped in an evanescent field with $\xi = 1.4\ \mu m$.
c) (3 points) Qualitatively describe a method based on the use of an additional laser beam with which (weak) confinementparallel to the surface could be provided to create a "pancake-shaped" trap for a 2D atom gas. What sign would the detuning of the laser (i.e. the laser frequency minus atomic transition frequency) need to have?
d) (6 points) What is the thermal de-Broglie wavelength $\Lambda$of the trapped gas (atomic mass m) at a temperature T ? For the case of a bosonic ^{133}Cs gas at T^* confined in a pancake area of $A = (30\ \mu m)^2$, give an rough estimate for the atom number needed to enter the quantum degenerate regime (assume a uniform planar density, and treat the system qualitatively as if it were 3D).

Breadth IV (Durst)

Many materials of current interest can be modeled as systems of electrons in reduced dimensions. Consider a free (non-interacting) gas of electrons in two dimensions. The electron energy spectrum, measured from the Fermi energy ε_F , is $\varepsilon(\boldsymbol{p}) = p^2/2m^* - \varepsilon_F$, where m^* is the effective electron mass. (In what follows, you may assume that ε_F is much larger than all other energy scales.)

a) (5 points) Calculate the density of states, $p(\omega)$, as a function of energy ω. Sketch it.

b) (5 points) Imagine that interactions have turned the system into a conventional s-wave superconductor. The superconducting order parameter, Δ, takes the same value at all points on the Fermi surface. The excitations of this system are fermionic (Bogoliubov) quasiparticles. Calculate and sketch the quasiparticle density of states.

c) (5 points) Imagine that the symmetry of the superconducting order parameter changes from s-wave (i.e. $\Delta(\boldsymbol{p}) = \Delta_0$) to d-wave (i.e. $\Delta(\boldsymbol{p}) = \Delta_0(\mathrm{p}) \cos(2\theta\mathrm{p})$). The gap now vanishes linearly at four points (nodes) on the Fermi surface. Explain qualitatively how the quasiparticle density of states would change. Sketch it.

d) (5 points) How can one distinguish experimentally between s-wave and d-wave superconductors?

Breadth V (Yahil)

Density structures in the universe cause velocity perturbations. Overdensities slow down the expansion relative to the Hubble expansion and underdensities speed it up. In this question we look at the limiting case of a spherical underdensity that has evacuated a whole region of space and seek to determine the velocity perturbation at the edge of the region. Far from the region, the expanding medium is homogeneous and unperturbed by the region, with cosmological parameters $\Omega_m = 1$ and $\Omega_\Lambda = 0$. Consider a galaxy just inside the edge of the region.

a) (5 points) Show that, barring the effects of other perturbations, the velocity of this galaxy is expected to be 50% higher than if the region was filled (had the same density as the background medium). [Hint: consider the quantity Ht, where H is the Hubble parameter at the position of the galaxy and t is the age of the universe.]

b) (5 points) Explain why the velocity perturbation is expected to be smaller if the background medium has $\Omega_m < 1$. You do not have to calculate the velocity precisely, but you must explain the dependence of the velocity on Ω_m.

c) (10 points) One might think that the velocity perturbation can be used to measure Ω_m. Cite two reasons why this method might not be a good method. The reasons may be theoretical, observational, or a combination of theoretical and observational arguments.

Breadth VI (Simon)

Consider an isolated and spherical molecular cloud in our galaxy. Its radius is 1 pc, and the molecular hydrogen density and temperature are initially constant throughout with values 10^4 cm^{-3} and 20 K, respectively.

a) (5 points) Derive an estimate for the length of time it would take the cloud to collapse by its self-gravity.

b) (5 points) As the cloud collapses, it heats and the pressure in its interior resists the collapse. Derive an estimate for the time it takes for a pressure pulse originating at the surface to reach the center of the cloud. Compare this estimate with that in (a); is the cloud stable against collapse?

c) (5 points) Derive expressions for the Jeans length and Jeans mass and state the criterion for stability against collapse.

d) (5 points) What is the observational evidence that molecular clouds are threaded by magnetic fields?

"Constants and Unit Conversions"

Avogadro number	$N_A = 6\ 10^{23}$ mol^{-1}
Boltzmann constant	k = 1.38 10^{-23} J/K
Planck's constant	h = 6.62 10^{-34} Js
Speed of light	c = 3 10^{8} m/s
	$\hbar c$ = 197 MeV fm
electron mass	m_e = 511 KeV/c^2 = 9.11 10^{-31} kg
electric charge	e = 1.6 10^{-19} C
fine structure constant	$\alpha = e^2/4\pi\ \varepsilon_0\ \hbar c$ = 1/137
Gravitational constant	G = 6.67 10^{11} m^3/kg s^2
Mass of Sun	M_o = 1.99 10^{30} kg
Radius of Sun	R_o = 6.97 10^{8} m

1 eV = 1.6 10^{-19} J
1 eV/c^2 = 1.78 10^{-36} kg
1 barn = 1 10^{-28} m^2