STONY BROOK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY

Placement Examination **Quantum Mechanics**, August 25, 2005

General instructions: Three problems are given. You should do any two. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used.

Quantum Mechanics I (Zahed)

We define the μ-neutrino and τ-neutrino states through the mixing angle θ.

$$|\nu_\mu\rangle = \cos(\theta)|\nu_1\rangle + \sin(\theta)|\nu_2\rangle$$
$$|\nu_\tau\rangle = -\sin(\theta)|\nu_1\rangle + \cos(\theta)|\nu_2\rangle$$

with $|\nu_{1,2}\rangle$ orthogonal and normalized neutrino states of energy $E_{1,2} = \sqrt{{p_{1,2}}^2 + {m_{1,2}}^2}$.

a) (10 points) Suppose $|\nu_\mu\rangle$ is produced at $t = 0$ in the upper atmosphere. What is the probability that at later time it is detected as $|\nu_\tau\rangle$?

b) (10 points) In 1998 Kamiokande in Japan detected a $\mu\rightarrow\tau$ neutrino oscillation produced in the atmosphere on the other side of the earth. If the neutrino energies are about 1 GeV, and the distance covered by the oscillation is about 1.3 10^4 km, give an estimate of the neutrino mass squared difference $\Delta m^2 = {m_2}^2 - {m_1}^2$ in $(\mathrm{eV})^2$.

Quantum Mechanics II (van Nieuwenhuizen)

a) (5 points) What is the ratio $E_0({}^4He^+)/E_0(H)$ of the energy of the lowest level of the once-ionized ${}^4He^+$ atom and the energy of the lowest level of the hydrogen atom? By what factor is this ratio modified if one takes reduced masses into account?

b) (5 points) The first 3 Bohr levels of ${}^4He^+$ (those with n=1 and n=2) contain each several states if one takes the angular momentum of the electron into account. Write down all 7 quantum numbers of these states (one line for each Bohr level).

c) (5 points) Which interactions produce the fine structure? Indicate in a figure how the fine structure shifts and splits these lines. Give a short explanation (no more than 4 lines).

d) (5 points) Does the spin of the nucleus affect the ground state energy of H and/or $^{4}He^{+}$? Mention two effects which further affect the spectral lines but which are not given by relativistic quantum mechanics.

<u>Quantum Mechanics III (Goldhaber)</u>

A *Hermitian* operator O satisfies $\langle \phi | O | \psi \rangle = \langle \psi | O | \phi \rangle^*$. A *self-adjoin* operator has a complete set of orthogonal normalized eigenstates, all with real eigenvalues.

a) (5 points) Show that a self-adjoint operator is automatically Hermitian.

b) (5 points) Consider the infinite square well, in which only the interval $x=0$ to $x=a$ is accessible. Impose the boundary condition that wave functions must vanish at $x=0,a$. Show that the Hamiltonian for this system is self-adjoint.

c) (5 points) Show that the operator $p = -i\hbar\partial_x$ is Hermitian, but NOT self-adjoint with the above boundary conditions.

d) (5 points) Now consider a particle constrained to move on a closed curve of length a. Impose the boundary condition $\psi(x=a) = \exp(i\alpha)\psi(x=0)$, $\psi'(x=a) = \exp(i\alpha)\psi'(x=0)$, for some specified value of the real constant α. With these conditions find the normalized eigenfunctions of the Hamiltonian, and show that this time p also is self-adjoint.