# *STONY BROOK UNIVERSITY*
# *DEPARTMENT OF PHYSICS AND ASTRONOMY*

Placement Examination **Statistical Mechanics**, August 25, 2005

**General instructions**: Three problems are given. You should do any two. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used.

## Statistical Mechanics I (Verbaarschot)

a) (5 points) What is an order parameter?
b) (5 points) Suppose that the potential for the order parameter is given by $V(\phi) = a(T - T_C)\phi^n + b\phi^{n+2}$. For what values of *a, b* and *n* does this potential describe a second order phase transition? (Consider only positive *n*.)
c) (5 points) For $T \to T_C$ the order parameter behaves as $\sim (T - T_C)^\beta$. Calculate $\beta$.
d) (5 points) Now add the term $g\phi$ to the potential. How does the order parameter behave as a function of *g* for $T \to T_C$?

## Statistical Mechanics II (Prakash)

Consider a 3 dimensional system of $N$ noninteracting spin 1/2 fermions of mass *m* and energy $\varepsilon_{\vec{p}} = p^2/(2m)$ ( $\vec{p}$ denotes the momentum and *p* is its magnitude) contained within a volume $V$. In equilibrium, the average occupation probability of a state with momentum $\vec{p}$ is given by the Fermi-Dirac distribution

$$\langle n \rangle = f(\varepsilon_{\vec{p}}) = \frac{1}{1 + \exp\left(\dfrac{\varepsilon_{\vec{p}} - \mu}{kT}\right)}$$

Above, $\mu$ is the chemical potential, *T* is the temperature, and *k* is the Boltzmann's constant.

a) (5 points) At $T = 0$, determine the relation between the number $N = \sum_{\vec{p}} \langle n_{\vec{p}} \rangle$ and the Fermi energy $\varepsilon_F$ (this is the energy of the largest momentum state $p_F$ ).

For temperatures satisfying $kT/\mu$ << 1, show that

b) (5 points) The chemical potential obeys the relation

$$\mu \approx \varepsilon_F \left[ 1 - \frac{\pi^2}{12} \left( \frac{kT}{\varepsilon_F} \right) + .... \right]$$

by considering leading order corrections due to the effects of temperature, and

c) (10 points) To leading order in $T$, the mean energy $U = \sum_{\vec{p}} \varepsilon_{\vec{p}} \langle n_{\vec{p}} \rangle$ is given by

$$U \approx \frac{3}{5} N \varepsilon_F \left[ 1 + \frac{5\pi^2}{12} \left( \frac{kT}{\varepsilon_F} \right) + ... \right].$$

**Hints:**

1. Use the replacement $\sum_{\vec{P}} \quad \rightarrow \frac{V}{\hbar^3} \int \frac{d^3 p}{(2\pi)^3}$.

2. Employ Sommerfeld's expansion

$$\int d\varepsilon \; X(\varepsilon) f(\varepsilon) \approx \int_0^\mu d\varepsilon \; X(\varepsilon) + \frac{\pi^2}{6} (kT)^2 \left( \left. \frac{dX}{d\varepsilon} \right|_{\varepsilon=\mu} \right) + .....$$

**<u>Statistical Mechanics III (Averin)</u>**

A large number $N >> 1$ of non-interacting bosons are confined to move in a one-dimensional quadratic potential producing harmonic oscillations of frequency $\omega$ for each particle. The system is in equilibrium at temperature $kT >> \hbar\omega$.

a) (10 points) Treating the system in the grand canonical ensemble, write down the equation for the chemical potential $\mu$ of this gas of bosons. Calculate $\mu$ from this equation replacing the sum over the oscillator states with an integral.
b) (5 points) From part (a), find the average occupation $n_0$ of the ground state of the oscillator and estimate the temperature range at which the gas of boson behaves as classical ideal gas.
c) (5 points) Compare the average occupations $n_0$ and $n_1$ of the ground and the first excited states and estimate the temperature at which $n_0$ becomes considerably larger than $n_1$.