

STONY BROOK UNIVERSITY**DEPARTMENT OF PHYSICS AND ASTRONOMY**

Comprehensive Examination, January 25, 2006

General instructions: Twelve problems are given. You should do any four, subject to the constraint that you must answer at least one from the "experiment" and one from "breadth" category. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the problem's number, e.g. Experiment II or Breadth I etc., and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used. You will find a list of useful constants at the end of the exam.

"Experiment"**Experiment I**

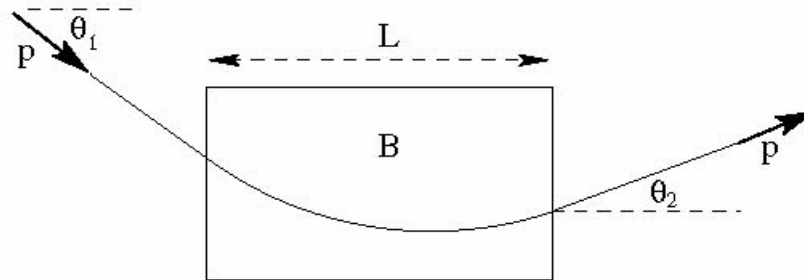
Consider the sequential decay of the K^* meson (a $d\bar{s}$ meson, spin $s = 1$): $K^* \rightarrow K^0\pi^0$, followed by the decays $K^0 \rightarrow \pi^+\pi^-$ and $\pi^0 \rightarrow \gamma\gamma$. The width (full width at half maximum) of the K^* is 50 MeV, the mean life of the K^0 (a $d\bar{s}$ meson, $s = 0$) is 0.89×10^{-10} s, and the mean life of the π^0 (a superposition of $d\bar{d}$ and $u\bar{u}$ states, $s = 0$) is 8×10^{-17} s.

- (4 points) Calculate the mean lifetime of the K^* .
- (4 points) Based on the life times and the decay modes, indicate the interaction that mediates the: (i) K^* decay, (ii) the K^0 decay, and (iii) the π^0 decay.
- (4 points) Suppose you have measured the momenta of the two charged pions from the K^0 decay as: $p_{\pi^+} = 350 \text{ MeV}/c$ and $p_{\pi^-} = 240 \text{ MeV}/c$, with a relative angle between the π 's of 90 degree. Calculate from these measurements the invariant mass of the $\pi^+\pi^-$ system.
- (4 points) Assume the momentum is obtained from the measurement of several points along the pion's trajectories in a magnetic field, e.g. using a drift chamber or multiwire proportional chambers filled with a argon gas mixture. List possible sources of uncertainty in the momentum measurement.
- (4 points) Calculate the energy loss dE/dx of a charged pion with a momentum of 500 MeV/c in Argon gas at STP ($Z = 18$, $A = 40$, density $\rho = 1.78 \text{ g/L}$).

Experiment II

We will investigate the measurement of momentum and energy of high energy electrons or muons via a magnetic spectrometer and a sampling calorimeter.

In a dipole magnet spectrometer the momentum of a charged particle is measured from the angular deflection traversing a constant magnetic field.



- a) (5 points) Consider a charged particle of momentum p incident on a region of magnetic field B at an angle θ_1 as shown in the figure. Assume the magnetic field is perpendicular to the particles trajectory. Derive the exact relation between the entrance angle θ_1 and the exit angle θ_2 . Note: Use counterclockwise as positive sense for θ_1 and θ_2 .
- b) (5 points) If the entrance and exit angles are each measured with a resolution $\delta\theta$, derive the relative momentum resolution $\delta p/p$ as a function of momentum. Here you may use a small angle approximation.

In a lead-scintillator sampling calorimeter particles interact in the lead layers. The layers are interspersed with scintillator layers that sample the energy deposited in the calorimeter.

- c) (5 points) List the principle energy loss mechanisms for high energy electrons and muons in matter, consider the energy range between 1 GeV and 100 GeV. Which, electron or muon, has the larger range in matter?
- d) (5 points) Derive a formula, up to a numerical constant, for the relative energy resolution $\delta E/E$ of a sampling calorimeter as a function of energy.

Experiment III

- a) (5 points) When a collimated beam of monochromatic, circularly polarized light passes through a quarter-wave plate, the beam is transformed into linearly polarized light. In a few short sentences, explain why this happens. What is the orientation of the linear polarization?
- b) (5 points) If a second, identically oriented quarter wave plate is placed in the beam after it passes through the first one, what is the polarization of the emerging beam? Explain how you got your answer.
- c) (5 points) If the second quarter wave plate is rotated to a 45° ($\pi/4$) angle with respect to the first wave plate, what is the polarization? Explain how you got your answer.
- d) (5 points) If the second quarter wave plate is replaced by a retroreflecting mirror, what is the polarization of the reflected beam? Explain how you got your answer.

Hint: You can exploit the Jones matrices to help solve these problems using $J_l \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

for a horizontal linear polarizer and $J_q \equiv \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ for a quarter wave plate. Note that $J(\theta) =$

$R(-\theta) J(0) R(\theta)$ where $R(\theta) \equiv \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$

Experiment IV

Consider a table-top X-ray diffraction experiment.

- a) (4 points) Draw a simple diagram showing how you can generate X-rays with up to 20 keV of energy.
- b) (4 points) Draw a graph of the spectrum you might get from this source if you use a copper target.
- c) (4 points) What device could you use to measure the X-rays? How does this device work?
- d) (4 points) How could you use this X-ray source and detector in conjunction with a LiF crystal to measure the lattice spacing of the crystal?
- e) (4 points) How could you also measure Planck's constant? Give a detailed answer with expressions justifying your answer. (4 points)

Experiment V

Astronomers have developed diagnostics at a variety of different wavelengths to calculate star-formation rates in galaxies. Because most galaxies are too distant, there is often insufficient data to make a rigorous assessment of star-forming regions contained in these galaxies, and thus these diagnostics are based on what is known about star formation in nearby galaxies.

- a) (4 points) What is the definition of the initial mass function (IMF), and why is it important for determining star formation rates?
- b) (4 points) The IMF is one of several parameters that factors into the estimation of star formation rates. Name at least two additional parameters.
- c) (9 points) Astronomers use measurements at the following wavelengths to estimate star-formation rates:
 - (i) ultraviolet luminosity,
 - (ii) hydrogen recombination line luminosity,
 - (iii) infrared luminosity.

State why measurements at these wavelength are useful for calculating star-formation rates. Give at least one advantage and one disadvantage of using each wavelength range.

- d) (3 points) Which of the above diagnostics would be adequate for estimating star formation rates in disks of spiral galaxies like our Galaxy, and in nuclear starbursts often associated with galaxy collisions? Provide reasons for your choices.

Experiment VI

The Sun at a distance $D = 1$ parsec (pc) subtends a diameter of about 10 milli-arcseconds (mas) and has a visual magnitude of $V \sim 0$ and blue minus visual magnitude of $B - V = 0.65$. An object in the halo of a distant galaxy, identified as a very bright globular cluster, also has a diameter of 10 mas.

- a) (2 points) The globular cluster has a $V = 25$ and $B - V = 0.65$ and is comprised of stars identical to the Sun. Calculate the beam “filling factor”, f , - i.e., the fraction of the globular cluster’s surface filled by stellar disks.
- b) (3 points) Derive an expression for f in terms of the number n of stars in the cluster and the distance d to the globular cluster.

Measurements of a few outlying stars lead to an estimate of the *rms* velocity of $\Delta v = 30$ km/s. Assuming all the stars are like the Sun, estimate

- c) (7 points) the distance d in parsec,
- d) (4 points) number of stars n and
- e) (4 points) the mean stellar density ($\#/pc^3$).

"Breadth"**Breadth I**

There is strong interest in the search for super-symmetric particles at the Large Hadron Collider (LHC), which is scheduled to begin operation in 2007. The LHC collides protons head-on with protons, each beam having an energy of 7 TeV. The lightest super-symmetric particle (LSP - with the lowest mass) is expected to be a point particle, which is electrically and color neutral. Assume the absolute conservation of R-parity, which distinguishes normal and super-symmetric particles (super-symmetric particles have $R = -1$, but normal particles have $R = +1$).

- a) (5 points) Why is the LSP expected to be absolutely stable?
- b) (5 points) Over what range in the mass of the LSP could it be produced at the LHC?
- c) (5 points) How could reactions involving the production of at least one LSP be detected at the LHC?
- d) (5 points) Why is the LSP considered to be a possible candidate for dark matter?

Breadth II

Quarks and gluons at low temperature are confined. The typical confining energy density in the vacuum is $B = (150 \text{ MeV})^4$ in units where 1 fm is $1/(200 \text{ MeV})$, or equivalently $B = 63 \text{ MeV/fm}^3$. Current experiments at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory are set to unravel the quark-gluon-plasma (QGP) state. To illustrate the phase change from a hadronic gas to a QGP, we will consider a simple two-phase model: A gas of massless pions ($I_\pi=1, S_\pi=0$) at low temperature, and at high temperature a QGP of massless quarks and gluons consisting of up quarks ($I_u = 1/2, S_u = 1/2$), down quarks ($I_d = 1/2, S_d = 1/2$), and gluons ($I_g = 0, S_g = 1$). The quarks come in three different colors, while the gluons come in 8 different colors. The notations are I for isospin and S for spin.

- (4points) Calculate the energy density and pressure of an ideal massless pion gas in terms of T .
- (4 points) Calculate the energy density and pressure of a massless quark-gluon gas in terms of T and B .
- (4points) Draw the pressure-temperature diagram and show why a phase transition from a pion gas to a quark-gluon gas is expected.
- (4 points) Find the critical temperature T_c in MeV for the transition.
- (4 points) What is the latent heat per unit volume L released during this transition in terms of B ?

Note: You may use $\int_0^\infty \frac{dx x^4}{e^x - 1} = \frac{\pi^4}{15}$ and $\int_0^\infty \frac{dx x^4}{e^x + 1} = \frac{7\pi^4}{120}$ to perform some of the integration.

Breadth III

Consider a nitrogen molecule, N_2 , with a mass of 28 amu. Each atom in the molecule has nuclear spin $I=1$.

- a) (2 points) What are the possible total nuclear spin values?
- b) (6 points) What are the consequences for the molecular rotational spectrum (i.e. what rotational energy states are allowed for the different possible nuclear spin states)?
- c) (12 points) How could you determine the bond length of a nitrogen molecule based on its rotational spectrum? Considering that the energy separation between rotational states $J=12$ and $J=14$ is 0.013 eV, what is the bond length?

Breadth IV

Modern semiconductor crystal growth techniques allow for the formation of nearly perfect layered structures which can trap electrons within a very narrow layer near the surface. In many cases such electrons may be considered as a gas of non-interacting Fermi particles which can only move within a 2D plane with some effective mass m . Neglecting electron spin and scattering:

- a) (5 points) Find the density ρ of states (per unit energy per unit area) of such particles.
- b) (1 point) Now let a magnetic field B be applied perpendicular to the plane of electron motion. What is the frequency ω_c of the classical cyclotron motion of electrons in the field?
- c) (1 point) The quantization of the cyclotron motion gives the energy level structure identical to that of a harmonic oscillator. What is the distance between these "Landau" levels?
- d) (3 points) Assuming that at $B \neq 0$ the results of (a) are still applicable for the *average* density of states, find the number of states (per unit area) available on each Landau level.
- e) (5 points) Derive the classical expression for the Hall resistance. Next, assuming that this expression may be applied in the quantum situation, find the Hall resistance in the case when electrons completely fill i Landau levels.
- f) (5 points) In a few sentences, discuss the relation between your final result and the quantum Hall effect.

Breadth V

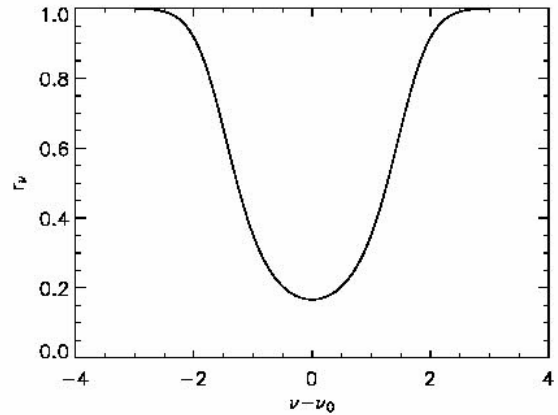
- a) (15 points) The value of Einstein's coefficient A of the neutral hydrogen ground state hyperfine transition at 21 cm wavelength is $2.85 \times 10^{-15} \text{ sec}^{-1}$. Describe quantitatively the physical conditions of hydrogen gas density and temperature required for this spectral line to be detected in emission. When these conditions are satisfied, derive the ratio of the populations of the upper and lower levels split by the hyperfine structure.
- b) (5 points) The wavelength of the $n = 171$ to 170 recombination line of neutral hydrogen is also about 21 cm and its Einstein A -value is 0.053 sec^{-1} (n is the principal quantum number). Describe qualitatively the physical conditions in which this line would arise. In two or three sentences (only!), explain why the transition probability of this line is so much greater than that of the line attributable the hyperfine interaction.

Breadth VI

In this problem you are asked to calculate the shape of a spectral line in the Schuster-Schwarzschild model, which assumes a scattering-dominated thin layer of atoms above a semi-infinite blackbody (continuum). You may assume that the transport equation for the radiation field in a semi-infinite plane parallel atmosphere is

$$\mu \frac{dI_\nu(\mu, \tau_\nu)}{d\tau_\nu} = I_\nu(\mu, \tau_\nu) - S_\nu(\mu, \tau_\nu)$$

Here I is the specific intensity, S is the source function, $\mu = \cos\theta$ is the angle measured relative to the normal to the atmosphere, the frequency is ν , and the optical depth is τ_ν . Assume scattering is coherent and isotropic.



- a) (6 points) Show that the source function both in the continuum I_c and in the line I_ν can be approximated by $S = J$. How is J defined in terms of I ?
- b) (6 points) In radiative equilibrium, the flux at each frequency F_ν and the continuum Flux F_c must be independent of optical depth. Solve the transport equation using the simplest approximations you can justify for $I_\nu(\mu = 1, \tau_\nu)$ and $I_c(\mu = 1, \tau)$, the outgoing intensities in the line and continuum normal to the atmosphere. You should express I_ν and I_c in terms of the fluxes F_ν and F_c .
- c) (6 points) If $I_\nu(1, \tau_\nu) = I_c(\mu = 1, 0)$, that is, if the line intensity on the base of the thin layer is the same as the emergent continuum intensity, show that the residual flux

$$r_\nu = \frac{F_\nu}{F_c} \approx (1 + \alpha\tau_\nu)^{-1}$$

where α constant of order of magnitude unity.

- d) (2 points) If the line is weak, evaluate the approximate line profile or shape (i.e., how does r_ν depend on the difference $\nu - \nu_0$, where ν_0 is the frequency of the center of the line).

"Constants and Unit Conversions"

Avogadro number	$N_A = 6 \cdot 10^{23} \text{ mol}^{-1}$
Boltzmann constant	$k = 1.38 \cdot 10^{-23} \text{ J/K}$
Planck's constant	$h = 6.62 \cdot 10^{-34} \text{ Js}$
Speed of light	$c = 3 \cdot 10^8 \text{ m/s}$ $\hbar c = 197 \text{ MeV fm}$
electron mass	$m_e = 511 \text{ KeV}/c^2 = 9.11 \cdot 10^{-31} \text{ kg}$
electric charge	$e = 1.6 \cdot 10^{-19} \text{ C}$
fine structure constant	$\alpha = e^2/4\pi \epsilon_0 \hbar c = 1/137$
π^\pm mass	$m_{\pi^\pm} = 139 \text{ MeV}/c^2$
K^* mass	$m_{K^*} = 896 \text{ MeV}/c^2$
K^0 mass	$m_{K^0} = 498 \text{ MeV}/c^2$
Gravitational constant	$G = 6.67 \cdot 10^{-11} \text{ m}^3/\text{kg s}^2$
Mass of Sun	$M_o = 1.99 \cdot 10^{30} \text{ kg} \sim 2 \times 10^{30} \text{ kg}$
Radius of Sun	$R_o = 6.97 \cdot 10^8 \text{ m} \sim 7 \times 10^5 \text{ km}$

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J}$$

$$1 \text{ eV}/c^2 = 1.78 \cdot 10^{-36} \text{ kg}$$

$$1 \text{ barn} = 1 \cdot 10^{-28} \text{ m}^2$$

$$1 \text{ radian} \sim 2 \times 10^5 \text{ arc seconds}$$

Bethe-Bloch Equation:

$$-\frac{dE}{dx} = 0.308 [\text{MeV cm}^2] \frac{zZ}{A} \frac{1}{\beta^2} \left(\ln \left(\frac{2m_e c^2}{Z \times 10 [\text{MeV}]} \beta^2 \gamma^2 \right) - \beta^2 \right)$$

z: projectile charge number

Z/A: charge to mass ratio of traversed matter

β : relativistic velocity

γ : Lorentz factor