STONY BROOK UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

Comprehensive Examination, September 7th, 2006

General instructions: Twelve problems are given. You should do any four, subject to the constraint that you must answer at least one from the "experiment" and one from "breadth" category. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the number of the problem and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used. You will find a list of useful constants at the end of the exam.

"Experiment"

Experiment I (Drees/Hobbs)

The Minimal Supersymmetric Standard Model predicts that every particle in the Standard Model has one or more partners distinguished by negative R parity. If these supersymmetric partners exist, it is likely that they will be discovered at the Large Hadron Collider (LHC). At the LHC there will be an active program to search for supersymmetric particles in proton-proton collisions with equal beam energies of 7 TeV.

Many of the supersymmetric particles that can be produced at the LHC have decay channels to highly energetic muons (or leptons in general but in this problem we only look at muons).

a) (3 points) What is the highest mass supersymmetric particle which could potentially be found at the LHC?
b) (3 points) Why must all decay chains finally contain one LSP (lightest supersymmetric particle)?
c) (3 points) How would you recognize this LSP in a detector?
d) (4 points) Name at least two resulting requirements for your detector to search for super symmetric particles?
e) (3 points) How do you identify muons, i.e. distinguish them from other charged particles?
f) (4 points) Which are the essential components of a detector system that can measure the momentum of a muon?

Solution:

a) Since supersymmetric particle and anti particle need to be created the largest mass is 7 TeV.

b) R parity is conserved, therefore the LSP must be stable.
c) Since LSP does not interact with ordinary matter it would be observed by missing energy.
d) Full solid angle coverage, calorimeter to measure energy.
e) Muons traverse material easily and can be detected after thick absorbers, e.g. after hadron calorimeter.
f) Magnetic field to bend muons, tracking chambers to measure trajectory in magnetic field.

Experiment II (Drees)

You are confronted with the following experimental observations:

- At low collision energies the reaction of protons with neutrons has a pronounced exit channel deuterium plus photon, i.e. p+n→ d+γ. This exit channel is not observed in p+p reactions.
- The lowest lying atomic energy level of deuterium (deuteron + electron) shows a hyperfine splitting into two levels.
- The deuteron has a magnetic moment of $\boldsymbol{\mu_D = 0.857\mu_N}$ ($\boldsymbol{\mu_N = \frac{e\hbar}{2m_p}}$), while the proton and neutron magnetic moments are $\mathbf{2.79\mu_N}$ and $\mathbf{-1.91\mu_N}$, respectively.

Use the first two observations to determine the quantum numbers - nuclear spin, isospin, parity - of the deuteron, a bound state of proton and neutron. Explain how you infer the quantum numbers from the observations.

a) (5 points) Isospin of the deuteron.

b) (5 points) Nuclear spin of the deuteron. (Assume there is no orbital angular momentum between p and n , i.e. L=0)

c) (5 points) Parity of the deuteron. (Do NOT use the assumption L=0)

Analyze the observed magnetic moment μ_D of the deuteron:

d) (5 points) Compare μ_D to the sum of μ_p and μ_n. Provide a possible explanation for the difference.

Solution:

a) Assume isospin T symmetry. With proton T=1/2 and T_3=1/2 and neutron T=1/2 and T_3=−1/2 one can construct a T=0 singlet with a np state and T=1 triplet with pp,np,nn states:

   Since only the np state is observed: T=0 and T_3=0

b) Nuclear spin J. With I, the angular momentum of electron, the total angular momentum F is F=I+J with possible magnitude values, which correspond to hyperfine levels:

$F=I+J,...,|I-J|$

If one assumes that there is no orbital angular momentum between proton and neutron (L=0) all nuclear spin results from the coupling of the proton and neutron spins. Thus there are two possibilities J=0 or J=1. If we assume the electron in the deuterium is in ground state, i.e. in the S-shell, the angular momentum of the electron is I=1/2.

For J=0 there is only one possible value of the total angular momentum F=1/2 and thus no hyperfine splitting. Observed are two levels and therefore the nuclear spin must be J=1.

c) Parity P: With the assumption of L=0 and the fact that proton and neutron have identical intrinsic parity it follow trivially that P=0. To avoid the L=0 assumption we can make use of symmetry arguments. The wavefunction ψ(r) may be factorized into a spacial contribution α, an isospin contribution β, and a spin contribution γ.

ψ(r)=α(r) beta(r) γ(r)

This wave function needs to be anti-symmetric under exchange of proton and neutron if we assume isospin symmetry. Since T=0 β(r) is a isospin singlet and thus anti-symmetric under p,n exchange. Contrary, the spin contribution γ(r) is a spin triplet since J=1 and thus symmetric. Since ψ(r) has to be anti-symmetric α(r) must be symmetric. On the other hand the symmetry of α is given by $(-1)^L$ and thus L must be even.

Finally, if L is even the parity must be P=+1.

d) Magnetic dipole moment: The sum of $\mu_p + \mu_n = 0.79\mu_N$ is smaller than the measured dipole moment $\mu_D = 0.875\mu_N$. The most plausible explanation is that the deuteron is not a pure L=0 state, but that there is also a L=2 contribution which will increase the dipole moment.

Experiment III (Weinacht)

A recent dramatic demonstration of matter wave interference (see e.g. O. Nairz, M. Arndt and A. Zeilinger, *Am. J. Phys*. **71**, 2003) was performed with C_{60} molecules. Interference was observed in a matter version of the famous Young's double slit experiment, originally performed with light.

a) (5 points) Draw a block diagram of an apparatus that you might use to observe a double slit diffraction pattern with C_{60} molecules. Label each element and describe its function.

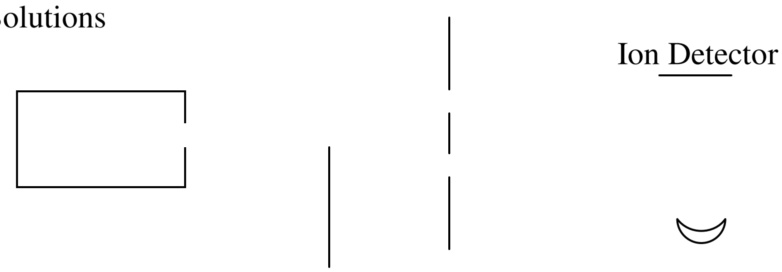
b) (5 points) What is the separation between interference maxima in the observation plane for a beam of molecules with an average velocity of 200 m/s and a slit separation of 100nm? How could you detect the molecules while resolving these fringes?

c) (5 points) In order to observe interference pattern, the experimentalists had to worry about the width of both the longitudinal velocity distribution as well as the transverse distribution. Discuss how each of these distributions affects the measurement, and how you could limit them. Be quantitative in your discussion.

d) (5 points) Discuss at least one way in which this experiment is different (in a measurable way) from an optical interference experiment.

Solutions

a/



C_{60} oven source   Collimation   Double slit   Focusing lens for ionizing molecules With laser beam

b/ $p=h/\lambda=mv$, $d\sin(\theta)=n\lambda$. Solving for the energy for a C_{60} molecule with a velocity of 200m/s yields $\lambda=2.76*10^{-12}$m. This yields the angle for the separation between maxima of $2.76*10^{-5}$ radians.

c/ Both longitudinal and transverse velocity distributions affect the measurement. If the transverse coherence length is smaller than the slit separation (because of a spread of transverse velocities in the beam), then the interference will not be visible. Essentially, the transverse velocity spread should be small enough such that the transverse deBroglie wavelength is larger than the slit separation. This means that $h/mv_{trans_max}>100$nm. This means that the spread of the transverse velocity distribution should be less than 0.0055m/s. This can be accomplished by using a narrow slit far from the source. Of course the price you pay is in the molecular flux.

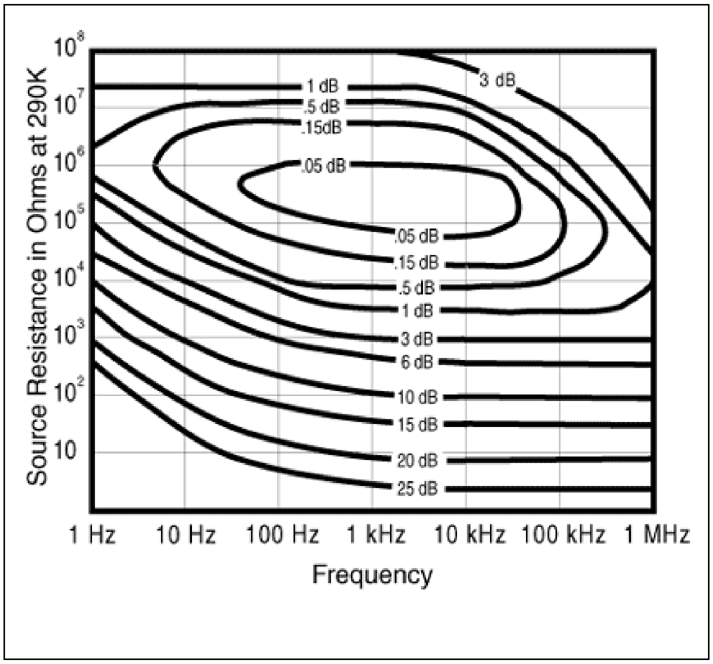
In terms of longitudinal coherence, the longitudinal coherence length must be larger than the path length difference between interference maxima for the fringes to be visible. The coherence length, $L \cong \lambda^2/\delta\lambda = \lambda v/\delta v$ should be larger than λ in order to see the first order fringe. In the experiment referenced above, a non velocity selected beam with $v/\delta v=0.6$ allowed the researchers to just be able to see the first order fringes. Velocity selection to $v/\delta v < 0.2$ allowed them to see second order fringes as well. Velocity selection can be accomplished with a pair of slotted rotating disks, which only pass a narrow group of molecules.

d/ Attraction of molecules to the slits, gravity, polarization of the optical beam but not the molecules, molecular orientation…

Experiment IV (Lukens)

Carbon resistors can have a resistance that varies roughly exponentially with temperature below ~4 K and are often used for very precise temperature measurements at low temperatures. Let 100 kΩ resistor R_x be measured using an equal-arm Wheatstone bridge. The bridge is driven using a variable amplitude 30kHz source. The null detector is a low noise amplifier having the noise figures shown below and a 100 MΩ input resistance.

Note: Refer all amplifier voltages in this problem are referred to the input (RTI), i.e. any voltage measure at the output is to be divided by the gain of the amplifier.



a) (4 points) What is the rms. output noise voltage of this amplifier (RTI) in a 0.1 Hz bandwidth around 10 kHz if the input is a 1 kΩ resistor? (Note that some parameters are different in this part of the problem.)

b) (4 points) If the bridge is driven by a 1 V_{rms} source, what is the output signal from the bridge if R_x is 1Ω greater than the other arms?

c) (8 points) If the entire bridge is cooled to 4.2 K and you are constrained to keep the power dissipated in R_x below 0.1 μW, what is the smallest change in R_x that can be detected using a 0.1 Hz bandwidth?

d) (4 points) If the resistance changes at the rate of 100% per Kelvin near 4.2 K, what is the smallest temperature change that can be detected for the conditions specified in part (c)?

Solution a:
In general the noise figure (NF) for an amplifier is

$$NF = 10\ \log\left(\frac{N_i + N_A}{N_i}\right),$$

where N_i is the part of the output power due to the thermal (Johnson) noise of the signal source at room temperature T_R, while N_A is the noise added by the amplifier. From the contours, NF = 3 db for the conditions given, so that

$$\frac{N_A}{N_i} + 1 = 10^{0.3} \approx 2.0,$$

i. e. $N_A \cong N_i$. Noise power is proportional to average voltage squared, so that in our case $\langle V_i^2 \rangle \cong \langle V_A^2 \rangle$. The Johnson noise of the input resistance $R_i = 10^3\ \Omega$ is

$$4k_B T_R R_i B \cong 4\times1.38\times10^{-23}\times290\times10^3\times0.1 \cong 1.6\times10^{-18}\ \mathrm{V}^2 \cong (1.3\ \mathrm{nV})^2,$$

Since the amplifier adds an equal noise (in terms of $\langle V^2 \rangle$), the total output noise, reduced to input, is $V_{\mathrm{rms}} = (2\times1.6\times10^{-18})^{1/2} \cong 1.8$ nV.

Solution b:
Let A and B are the two voltage terminals for the null detector and R_x is in leg B (see Fig. on the right). If $R_x = R + \delta R$, then

$$V_B = \frac{R + \delta R_x}{2R + \delta R_x} V \cong \frac{V}{2}\left(1 + \frac{\delta R_x}{2R}\right),$$

B A

while V_A does not change, so that the output voltage changes by

$$\delta(V_B - V_A) \cong \frac{V}{4}\frac{\delta R_x}{R}.$$

For our parameters, the change equals 2.5 $\mu\mathrm{V}_{\mathrm{rms}}$.

Solution c:
It is customary to define $(\delta R_x)_{\min}$ as the value of δR_x for which the r.m.s. signal voltage at the amplifier output equals the r.m.s. noise. Let us first calculate the total noise reduced to the amplifier input. (This excluded the amplifier gain from consideration.) From the noise figure plot, NF $\cong$ 0.05 db, so that $N_A/N_i \cong 10^{0.005}-1 \cong 0.011$. Hence the amplifier contribution N_A is just 1% of what the Johnson noise of the input (bridge) resistance R would be at T_R = 290 K. However, in our case the resistance is cooled to T_L = 4.2 K, so that the amplifier contribution is quite comparable with that of the source noise:

$$\langle V^2 \rangle_n = 4k_B(T_L + 0.011\ T_R)RB \cong 4\times1.38\times10^{-23}\times(4.2+0.011\times290)\times10^5\times0.1 \cong 4.1\times10^{-18}\ \mathrm{V}^2.$$

Considering the useful signal, the power dissipated in R_x is one quarter of the power in the bridge as a whole, i.e. $\langle V^2 \rangle / 4R = V_{rms}^2/4R$, so that the specified power limit $P_{\max} = 10^{-7}$ W means $\langle V^2 \rangle \le 4RP_{\max}$ = 4×10^5×10^{-7} ≅ 4×10^{-2} V^2. Using the result of part (b), we get that with such a drive voltage, the square of the signal voltage across the bridge is

$$\langle V^2 \rangle_s = \langle V^2 \rangle \frac{(\delta R_x)^2}{16R^2} \le 4\times10^{-2}\,\frac{(\delta R_x)^2}{16\times10^{10}}\ \mathrm{V}^2.$$

Now, requiring that $\langle V^2 \rangle_s = \langle V^2 \rangle_n$, we see that we can detect a resistance change of $\delta R_x \ge 4$ mΩ.

Solution d:

For small variations, $\frac{\delta R_x}{R} = \frac{\delta T}{T}$. With the resistance sensitivity found in part (c), this gives the minimum detectable temperature change to be $\delta T \ge 4.2\times4\times10^{-3}/10^5 \cong 0.2\ \mu$K.

<u>Experiment V (Walter)</u>

The Chandra X-ray observatory operates in the 0.25-10 keV regime. The mean effective area of the telescope is about 1000 cm^2. You detect an object with a flux of $3.6x10^{12}$ erg cm^{-2} s^{-1}. The typical photon energy is 2 keV.

a) (4 points) What is the count rate? How long will you have to integrate to achieve a S/N of 100 for the detection? The background is 10^{-5} photons $arcseconds^{-2}$ sec^{-1}. The pixel size is 0.5x0.5 arcsec. The spatial resolution is 0.5 arcseconds.
b) (4 points) Optical follow-up show the distance to the object to be about 500 pc. What is the intrinsic X-ray luminosity?
c) (4 points) Upon analyzing the spectrum, you detect the Fe XXVI Kα line in emission. Ten percent of the flux is in this line. What is the approximate energy of this line [Fe has $Z = 26$; the wavelength of H Lyα is 1216 angstroms]? Neglect relativistic effects. What is the S/N of the detection of this line?
d) (4 points) The Fe K line is formed at a temperature of ~ 10^8 K. What is the thermal width of this line? Is it resolvable in the ACIS-S spectrograph, with $E / \Delta E$ ~ 50. What resolution would you need to determine the temperature from the line width?
e) (4 points) The observation was long enough to collect 500 counts. You bin the data into 5 equal length intervals, that contain [65, 90, 100, 110, 135] photons. Is the source variable? At what level of significance can you reject the hypothesis that the source is constant? (See the provided graph of χ^2 values.)

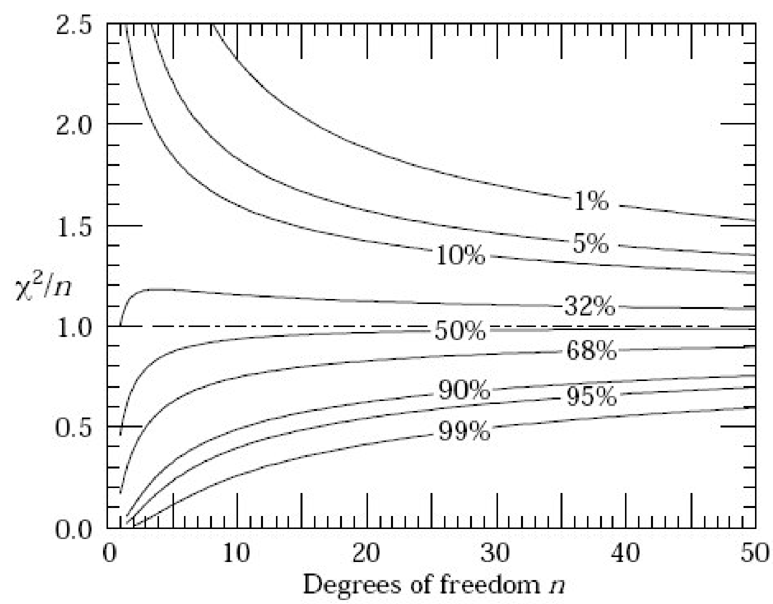


Figure 32.2: The 'reduced' χ^2, equal to χ^2/n, for n degrees of freedom. The curves show as a function of n the χ^2/n that corresponds to a given p-value.

<u>Experiment VI (Zingale)</u>

Parker (1958) predicted the existence of the solar wind and subsequently produced a steady-state, spherical model for it. Here, we derive an expression for the Mach number of a spherical wind as a function of radius. Consider the one-dimensional Euler equations for mass and momentum conservation describing fluid flow in spherical coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho u\right) = 0$$
$$\frac{\partial (\rho u)}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho u^2\right) + \frac{\partial p}{\partial r} = -\frac{Gm}{r^2}\rho$$

Here, ρ is the mass density, u is the fluid velocity (the speed of the solar wind), p is the gas pressure, G is the gravitational constant, and m is the mass of the Sun. If we assume an isothermal gas, with the equation of state

$$p = \rho c^2$$

where the sound speed, c, constant, then there is no need for an energy equation.

a) (8 points) Assuming that the flow is in a steady state, show that an ordinary differential equation describing the wind velocity squared, u^2, is:
$$\frac{1}{2}\left(1 - \frac{c^2}{u^2}\right)\frac{du^2}{dr} = -\frac{Gm}{r^2} + \frac{2c^2}{r}$$

b) (6 points) What is the critical radius, r_c, at which the flow can be sonic $(u = c)$?

c) (6 points) Introduce a dimensionless radius, $\xi = r/r_c$ and rewrite the ODE in terms of Mach number squared, M^2, and ξ, where $M = u/c$.

   This expression can be integrated to give an implicit equation for M^2 as a function of ξ. There are different classes of solution, depending on the value of the integration constant. Observationally, we know the wind persists for great distances from the Sun, and passes through $\xi = 1$. Consider the structure of the ODE and sketch the solution as $\xi \to 1$. Sketch the different types of behavior that solutions that can pass through $\xi = 1$ can have.

Parker argued that the boundary conditions for the Corona of the Sun expanding into vacuum are that $u << c$ as $\xi \to 0$, and that the hydrostatic pressure must vanish at $\xi \to \infty$. The latter constraint can be shown to imply that the solar wind is supersonic at large distances from the Sun. This is precisely what is observed. It is interesting to note that this same process can be used to consider steady spherical accretion onto a star (Bondi accretion; Bondi 1952). There, we arrive at the same equation for M as a function of ξ, but our boundary conditions are different.

Solution

a. In steady state, there is no time dependence for any of the fluid quantities, so the mass and momentum equations reduce to:

$$\frac{1}{r^2}\frac{d}{dr}(r^2\rho u) = 0 \tag{5}$$

$$\frac{1}{r^2}\frac{d}{dr}(r^2\rho u^2) + \frac{dp}{dr} = -\frac{Gm}{r^2}\rho \tag{6}$$

Using the continuity equation, we can rewrite Eq. 6 by expanding out the derivative in the first term and dividing by ρ as:

$$u\frac{du}{dr} + \frac{1}{\rho}\frac{dp}{dr} = -\frac{Gm}{r} \tag{7}$$

Our equation of state tells us that $dp/dr = c^2 d\rho/dr$, giving

$$u\frac{du}{dr} + \frac{c^2}{\rho}\frac{d\rho}{dr} = -\frac{Gm}{r} \tag{8}$$

Expanding out the derivative in the continuity equation (Eq. 5) yields

$$\frac{2}{r} + \frac{1}{\rho}\frac{d\rho}{dr} + \frac{1}{u}\frac{du}{dr} = 0 \tag{9}$$

which allows us to replace eliminate the density from Eq. 8, giving:

$$u\frac{du}{dr} + c^2\left(-\frac{2}{r} - \frac{1}{u}\frac{du}{dr}\right) = -\frac{Gm}{r^2} \tag{10}$$

Collecting the velocity derivatives, we have:

$$\left(1 - \frac{c^2}{u^2}\right) u\frac{du}{dr} = -\frac{Gm}{r^2} + \frac{2c^2}{r} \tag{11}$$

or

$$\frac{1}{2}\left(1 - \frac{c^2}{u^2}\right) \frac{du^2}{dr} = -\frac{Gm}{r^2} + \frac{2c^2}{r} \tag{12}$$

b. If $u = c$, then the lefthand side of Eq. 12 is 0, so the righthand side must be too. This gives

$$\frac{Gm}{r_c^2} = \frac{2c^2}{r_c} \tag{13}$$

or

$$r_c = \frac{Gm}{2c^2} \tag{14}$$

"Breadth"

Breadth I (Kuo)

Part A:

Consider symmetric nuclear matter like in the interior of a large nucleus that consists of an equal number of protons and neutrons.

a) (3 points) What are the binding energy per nucleon (BE/A) and density (ρ_0) of this matter at saturation?

At very high temperature and density there is a phase transition to a new state of matter, referred to as quark gluon plasma, in which the quarks and gluons are no longer bound to nucleons.

b) (3 points) What are the critical temperature and density approximately?

Part B:

Over the past years exotic nuclei with large neutron excess have been extensively studied at radioactive beam accelerators. Consider the exotic nucleus *He*(A=10,Z=2) and treat it as composed of 6 active neutrons confined in the shell model orbits ($0p_{3/2}$, $0p_{1/2}$, $0d_{5/2}$, $0d_{3/2}$,$0d_{1/2}$) outside an inert ^4He core.

c) (7 points) The wave function of the low-lying $J^{\pi} = 3^-$ excited states of this nucleus may be written as a linear combination of one-particle one-hole (*ph*) shell model excitations. Which *ph* excitations must be included in the above 3^- state?

d) (7 points) Suppose we use a separable interaction so that the particle-hole matrix element is $\langle ph|V|p'h'\rangle = -D(ph)^* D(p'h')$. The energy gap between the $0p$ and the $0d1s$ shell is $\hbar\omega$. Calculate the excitation energy of the 3^- states. Express your answer in terms of $\hbar\omega$ and *D*.

Breadth II (Sterman)

The "strange" meson K^- has a mass of about 500 MeV/c^2, while the π^0 meson has a mass of about 135 MeV/c^2. A strange quark has a charge of -1/3.

a) (6 points) In the quark model, what are the constituents of the K^- and π^0 mesons?

b) (8 points) What basic quark decay processes transforms the constituents of the K^- meson into constituents of the π^0? What leptons can be produced in this decay? Draw the Feynman diagrams!

c) (6 points) In the center-of-mass of the K^- meson, what is the maximum value of the π^0 energy that results from the decay? [Neglect lepton masses here.]

Breadth III (Goldman)

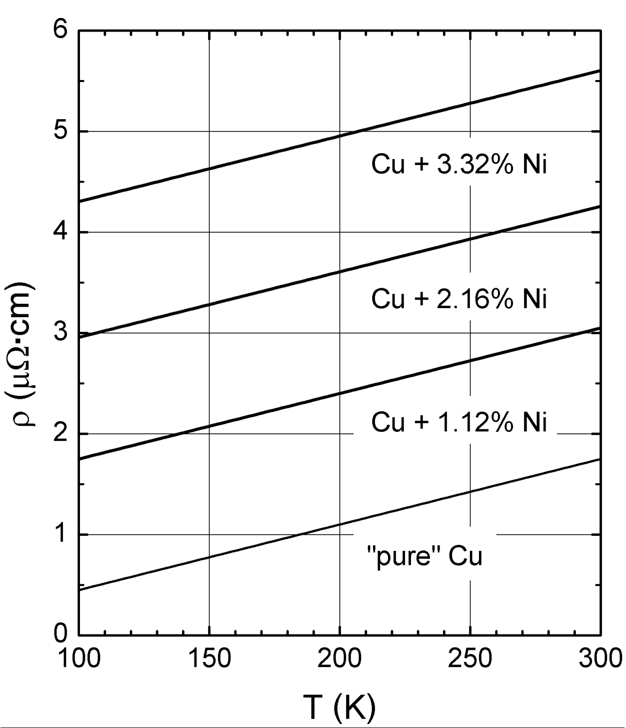
The data in the Figure below shows that resistivity of a metal obeys the "Matthieson rule": contributions from different scattering mechanisms are approximately additive. That is, the contribution due to electron scattering by impurity atoms (Ni here) is additive to the finite-temperature electron-phonon scattering, present even in "pure" Cu. In addition, the impurity scattering is approximately proportional to the impurity concentration in solid solution. Assuming random distribution of the impurity,

a) (14 points) estimate λ_i - the mean free path due to the scattering of electrons on Ni impurities for 1 atomic per cent of Ni impurity in Cu;

b) (6 points) find the effective scattering cross section of electrons on Ni impurity in Cu.

You may need some of the following table data.
Copper: atomic mass A = 63.5, atomic number Z = 29, mass density d = 8.96 g/cm^3. Nickel: atomic mass A = 58.7, atomic number Z = 28, mass density of 8.90 g/cm^3. Each Cu atom contributes 1 conduction electron with band mass equal to the vacuum mass $m_e = 9.1\times10^{-31}$ kg.

Hint: Use free Fermi gas model for conduction electrons in Cu, and the classical Drude model for electrical conductivity with Fermi velocity.



Solution:

Conduction electron density is equal to Cu atom number density: $d/n = A/N_A$; this gives n = 8.5×10^{22} cm^{-3} = 8.5×10^{28} m^{-3}.

Define: density of impurity atoms n_I << n. From the Figure, $\delta\rho = 1.3\times10^{-8}$ Ω·m for n_I = 0.0112 n (1.12 atomic % of Ni in Cu).

(a) Resistivity due to impurity scattering $\rho_i = 1/\sigma_i = m/ne^2\tau_i$, where the mean scattering time on impurities $\tau_i = \lambda_i / v_F$. Fermi velocity $v_F = \sqrt{2E_F/m} = \hbar k_F/m = (\hbar/m)(3\pi^2 n)^{1/3}$ = 1.6×10^6 m/s. Thus

$$\lambda_i = \frac{mv_F}{ne^2\rho_i} = 5.1\times10^{-8} \text{ m for 1.12\% Ni.}$$

(b) Electron-impurity scattering cross section $\Sigma_i = 1/(n_I\lambda_i) = 1/(0.0112\cdot n\lambda_i) = 2.0\times10^{-20}$ m^2. (If impurities are spherical, the electron moving with v_F sees the "impurity diameter" of 1.6 Å)

Breadth IV (Metcalf)

The first excited states of neutral ^{4}He have one electron in the ground state with $n = 1$ and the other electron in the next level with $n = 2$. These are called the "$n = 2$ states" even though one electron has $n = 1$.

a) (4 points) Enumerate the possible values of the quantum numbers for the various electronic angular momenta of these states.

b) (4 points) Enumerate the possible quantum numbers of the projection of these angular momenta on an externally imposed z-axis, for example, by a magnetic field. If you think this is more easily done by a diagram, draw such a diagram.

c) (4 points) Finding the wave function for such states as in part (1) above begins with a product of the wave function for each electron, say $|n_1, l_1, m_{l1}\rangle\,|\,s_1,m_{s1}\rangle\,|n_2, l_2, m_{l2}\rangle\,|\,s_2,m_{s2}\rangle$. However, this violates the Pauli principle even though the two electrons have different principal quantum numbers n_1 and n_2. Explain why. What can be done to make a wave function that satisfies the Pauli principle?

d) (4 points) Draw an energy level diagram of the states in part 1 above and also the ground state. Indicate the allowed and forbidden transitions between these $n = 2$ states and the ground state in which both electrons have $n = 1$, and provide an explanation of why some are forbidden.

e) (4 points) Some but not all of these $n = 2$ states have fine structure. Which ones, and why not the others? In the course of answering this, explain what the origin of fine structure is.

Answer:
1. $n = 2$ for all, ` could be zero or 1, and the spins could combine to be $s = 0; 1$. Thus there are four states all together. The $s = 0$'s are called singlets, and there are two of them, each with ` = 0; 1 and likewise for the $s = 1$'s that are called triplets.
2. Each orbital and spin quantum number combine to make a total J, and each J state has $2J+1$ Zeeman
sublevels.
3. The product state does not have the necessary exchange symmetry required by the Pauli principle. Some more suitable wave functions would be *fjn1; `1; m`1i ¡ jn2; `2; m`2igfjs1; ms1i + js2; ms2ig* or
fjn1; `1; m`1i + jn2; `2; m`2igfjs1;ms1i ¡ js2; ms2ig.
4. S states cannot couple to the ground state because it's also an S state. Moreover, the 3P state can't
couple to the ground state because singlet-triplet coupling is also higher order. So only the singlet *P* state can make allowed transitions to the ground state.
5. Fine structure originates from the interaction of the electron's intrinsic magnetic moment = 2 1B with the magnetic field that arises from the orbital motion. For ` = 0 states this must vanish, and for the singlet P state there is no net contribution from the intrinsic magnetic moments of the two electrons because they're oppositely aligned. So only the triplet P state has spin orbit or fine structure splitting, and the consequence is three states having $J = 0; 1;$ and 2.

Breadth V (Lattimer)

(20 points) A double line spectroscopic binary is observed to have two distinct sets of spectral lines that oscillate redward and blueward with a period of 34 days. The set of lines from the brighter star have a maximum relative displacement of about 0.004% while the other set has a maximum relative displacement of about 0.006%. The spectral lines from the dimmer star are what is expected from a white dwarf. In addition, the total flux observed from the system is periodic, with a period equal to the spectral line shift period. Determine the masses of the components.

Solution

Kepler's law for a binary is

$$G\frac{M_1 + M_2}{a^3} = \left(\frac{2\pi}{P}\right)^3$$

where M_1 and M_2 are the component masses, a is the semi-major axis and P is the orbital period. We also have

$$M_1 a_1 = M_2 a_2, \qquad a_1 + a_2 = a.$$

The maximum observed orbital velocity from component i is

$$v_{obs,i} = \frac{2\pi}{P} a_i \sin i = \Delta_{i,max} c$$

where i is the inclination angle of the binary to the line of sight and $\Delta_{i,max}$ refers to the maximum relative Doppler shift of star i. Thus

$$\frac{M_1}{M_2} = \frac{v_{obs,2}}{v_{obs,1}} = \frac{\Delta_{2,max}}{\Delta_{1,max}} = 1.5.$$

Also,

$$\frac{(M_2 \sin i)^3}{(M_1 + M_2)^2} = \frac{P}{2\pi G} v_{obs,1}^3.$$

If we use units where masses are in solar masses, time in years and distances in astronomical units, we can set $G = 1$. We have

$$v_{obs,1} = .00004 \times 3 \cdot 10^{10} \ \text{cm/s} \times \frac{3.1 \cdot 10^7 \ \text{s/yr}}{1.5 \cdot 10^{13} \ \text{cm/AU}} = 2.48 \ \text{AU/yr}.$$

The inclination angle is unknown, but the fact that the system is observed to eclipse indicates it is near 90 degrees. Setting $\sin i = 1$ one finds

$$M_2 = \left(\frac{5}{2}\right)^2 \frac{P}{2\pi} v_{obs,1}^3 = 1.41 \ \text{M}_\odot$$

and $M_1 = 1.5 M_2 = 2.1 \ \text{M}_\odot$.

Breadth VI (Evans)

Ultraluminous infrared galaxy mergers are observed to have total luminosities of $\geq 10^{12}$ solar luminosities (L_o). Up to 99% of the luminosity from these colliding galaxies is detected at far-infrared wavelengths – the radiation is thermal emission from dust heated by imbedded, active star formation (a starburst), an active galactic nuclei (mass accretion onto a super-massive black hole), or a combination of both. In this problem, we compare the relative gas consumption rates of an active galactic nucleus (a & b) and star formation (c & d).

a) (10 points) The standard condition for accretion of material onto a super-massive black hole is that the outward radiative force of the accretion disk near the black hole must be less than the inward gravitational force; in the case where the forces are equal, the luminosity of the radiation is referred to as the Eddington Luminosity, L_{edd}. Derive an expression for the mass of the black hole, M_{BH}, by making the assumption that the radiative and gravitational forces are exactly balanced. Express the mass in terms of the Eddington Luminosity and the Thompson scattering cross section, σ_T. What is the mass of a black hole (in solar masses) associated in with an active nucleus that has an Eddington Luminosity of 1×10^{12} L_o?

b) (3 points) If 10% of the accreting flow is converted to radiative energy, what is the mass accretion rate of a black hole with a $L_{edd} = 10^{12}$ L_o?

c) (2 points) Stars form from molecular gas. Once formed, stars emit energy during most of their lifetime via fusion of hydrogen into helium – this process has a 0.7% efficiency in terms of the conversion of mass to energy, and it continues until 10% of the stellar mass is converted from hydrogen to helium. Given this, write down an expression for the luminosity of a star, L_*. This expression should include the mass of the star, M_*, and the lifetime of the star, τ, in its hydrogen 'burning' phase.

d) (5 points) For starbursts, 5% of the total stellar mass is converted from hydrogen to helium in 10^8 years. If the stellar population produces a total luminosity of 10^{12} L_o, what is the star formation rate (in solar masses per year)? You can assume that the luminosity, and thus the star formation rate, is constant over the lifetime of the burst. If the burst lasts 10^8 years, how much molecular gas is needed? Finally, how does this rate of gas consumption compare with that of an active galactic nucleus of the same luminosity?

"Constants and Unit Conversions"

Avogadro number	$N_A = 6\ 10^{23}$ mol^{-1}
Boltzmann constant	$k = 1.38\ 10^{-23}$ J/K
Planck's constant	$h = 6.62\ 10^{-34}$ Js
Speed of light	$c = 3\ 10^{8}$ m/s
	$\hbar c = 197$ MeV fm
Electron mass	$m_e = 511$ KeV/$c^2 = 9.11\ 10^{-31}$ kg
Electric charge	$e = 1.6\ 10^{-19}$ C
Fine structure constant	$\alpha = e^2/4\pi\ \varepsilon_0\ \hbar c = 1/137$
Gravitational constant	$G = 6.67\ 10^{11}$ m^3/kg s^2
Mass of Sun	$M_o = 1.99\ 10^{30}$ kg
Radius of Sun	$R_o = 6.97\ 10^{8}$ m
Solar Luminosity	$L_o = 3.90 \times 10^{26}$ W
Thomson scattering cross section	$\sigma_T = 6.65 \times 10^{-29}$ m2

1 pc = 3.09×10^{16} m
1 eV = $1.6\ 10^{-19}$ J
1 eV/c^2 = $1.78\ 10^{-36}$ kg
1 barn = $1\ 10^{-28}$ m^2