STONY BROOK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY

Placement Examination **Quantum Mechanics**, September 1, 2006

General instructions: Three problems are given. You should do any two. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctors approval a foreign language dictionary. No other materials may be used.

Quantum Mechanics I

Consider spinless electrons in 3 dimensions moving in a constant magnetic field $\vec{\boldsymbol{B}} = \boldsymbol{B}_z$.

a) (10 points) Find the electron energy levels.
b) (5 points) What is the degeneracy of each level for fixed z-momentum?
c) (5 points) Write down the general form of the wave-function.

Note: Use the vector potential associated with $\vec{\boldsymbol{B}}$ in the Landau gauge $\vec{\boldsymbol{A}} = \boldsymbol{xB\hat{y}}$.

Quantum Mechanics II

As a first approximation, the neutron-proton force can be described by an attractive square well potential

$$V(r) = -V_0\Theta(a - r)$$

where Θ is a step function. The range a of the force is of the order of the pion Compton wavelength $\lambda_\pi = \hbar/(m_\pi c)$ with $m_\pi = 139$ MeV in units where $\hbar c = 197$ MeV fm.

a) (10 points) What is the minimum depth of the potential for a bound state to form?
b) (4 points) What lower bound can be put on V_0 if there were no bound proton-neutron (Deuteron) state?
c) (2 points) Derive a precise relation between E and V_0, where E is the binding energy of the Deuteron.
d) (4 points) Improve your estimate in b) for V_0 by using the fact that $E = -2.22$ MeV.

Note: Use a proton mass equal to a neutron mass, $m_N = m_P = 940$ MeV.

Quantum Mechanics III

The electron neutrino and τ neutrino mixing in matter is described by the Hamiltonian:

$$H = -\omega\,(\cos(2\theta)\,\sigma_3 - \sin(2\theta)\,\sigma_1) + \frac{GN}{\sqrt{2}}\sigma_3$$

$\sigma_{1,3}$ are Pauli matrices

$\omega = (m_\tau^2 - m_e^2)/2E = \Delta\, m^2/2E$

θ vacuum mixing angle

G Fermi decay constant

N the electron density in matter

E the relativistic neutrino energy

a) (4 points) Find the eigenvalues of *H*.
b) (6 points) Find the eigenvectors of *H* as linear combinations of the free electron and tau neutrino states, ie

$$|\nu_1 >= +\cos(\theta_m)|\nu_e > +\sin(\theta_m)|\nu_\tau >$$

$$|\nu_2 >= -\sin(\theta_m)|\nu_e > +\cos(\theta_m)|\nu_\tau >$$

θ_m is the mixing angle in matter. Give its explicit dependence on θ, ω, G, N.

c) (4 points) If the electron density in matter *N* is constant, what is the probability for an electron neutrino to turn into a tau neutrino in matter?
d) (2 points) Find N for which this mixing is maximum
e) (4 points) At what distance $L = ct$ would all electron neutrinos entering matter be converted to tau neutrinos?