STONY BROOK UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

Comprehensive Examination, September 5, 2007

General Instructions: Twelve problems are given. You should do any four, subject to the constraint that you must answer at least one from the experiment and one from the breadth category.

If you do more than four problems, you must decide which four to submit. Do not hand in any more than four solutions!

Each problem counts 20 points, and the solution should take on the order of 45 minutes. Use one exam book for each problem, and label it carefully with the problem designation and your name. You may use a one page help sheet, a calculator, and with the proctor's approval, a foreign language dictionary. No other materials may be used.

Many problems spill onto two pages. **Make sure you answer all parts of each question**

Atomic, Molecular, and Optical, Breadth

The non-relativistic Schrödinger equation for the Hydrogen atom without spin yields the same energies as the Bohr model. Two important spin corrections to these energies are due to coupling between the electron spin, S, and orbital angular momentum, L, to yield J, as well as coupling between J and the nuclear spin, I to yield a total angular momentum F.

a) (3 points) Give the allowed energies for the Hydrogen atom based on the non-relativistic Schrödinger equation. Calculate the wavelength for Lyman α ($n = 2 \rightarrow 1$) radiation.

b) (14 points) Calculate the spin-orbit interaction energy to within an order of magnitude for an n=2, l=1 solution to the Schrödinger equation. Hint: for solutions of the radial Schrödinger equation,

$$\langle \frac{1}{r^3} \rangle = \frac{Z^3}{n^3 a_0^3 l(l+1)(l+1/2)}$$

c) (3 points) Is the energy splitting due to the interaction of the nuclear magnetic moment with the magnetic field of the electron larger or smaller than the spin-orbit splitting? Explain.

Constants you might find relevant are
$a_0 = 0.5\times10^{-10}$m, μ_B=9.27×10^{-24}Am^2, μ_N=5.05×10^{-27}Am^2, ϵ_0=8.85×10^{-12}AsV^{-1}m^{-1}.

Condensed Matter Breadth

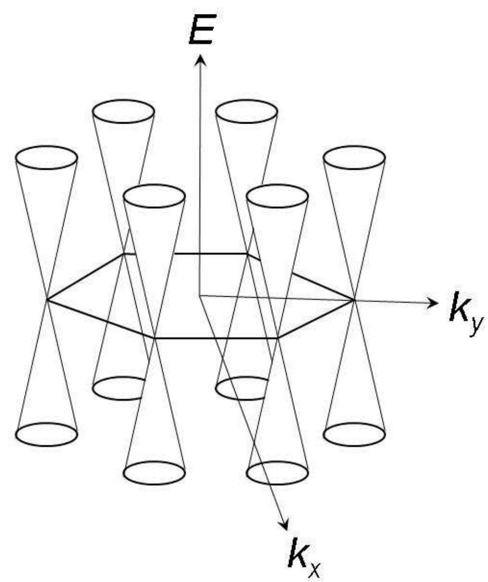
A few years ago a process was invented for isolating a single layer, one atom thick, of crystalline graphite. This is carbon in a honeycomb lattice with perfect triangular symmetry; a single sheet is referred to as "graphene."

a) (4 points) Sketch the honeycomb graphene crystal structure. Let the distance between carbon atoms be called a. Show the primitive unit cell and choose appropriate primitive translation vectors $\vec{a} = (a_x, a_y)$ and $\vec{b} = (b_x, b_y)$. How many carbon atoms are in the primitive unit cell?

b) (4 points) Find the primitive translation vectors $\vec{G}_1$ and $\vec{G}_2$ of the reciprocal lattice. Make a drawing of the reciprocal lattice (using the same x and y Cartesian directions that you used for the honeycomb direct lattice.) Show the first Brillouin zone, being careful to get its orientation correct with respect to the lattice you sketched in part a).

c) (4 points) Explain why the six corners of the Brillouin zone are not equivalent, but rather occur in two families of three corners each.

The picture to the right shows the electron dispersion relation $E(\vec{k})$ relation expected in graphene, with energy E shown vertically, and the two directions k_x and k_y of $\vec{k}$-space shown "horizontally." (The hexagon is the first Brillouin zone which you should have found in part b.) The energy versus $\vec{k}$ has the unusual form $E(\vec{k}) = \pm\hbar v_F|\vec{k}|$ in the vicinity of the six Brillouin zone corners, provided the origin of $\vec{k}$ is taken at the corner. This is what the double cones are depicting – energy is rising linearly with $|\vec{k}|$ in all directions above $E = 0$, and also falling linearly with $|\vec{k}|$ in all directions below $E = 0$. The Fermi level is at $E = 0$ when graphene is undoped. The Fermi velocity v_F is about 10^8 cm/s, independent of doping. An elegant method to produce carriers in the graphene sheet is to apply a "gate voltage" between the substrate and the graphene layer.



d) (4 points) Give an expression for the density of electron states of graphene for $|E|$ not too far from $E = 0$ *i.e.*, $|E| < 0.2$ eV.

e) (4 points) Suppose you could measure the electronic specific heat $C(T)$ in the temperature interval $0.1\text{K} < T < 10\text{K}$, for a range of gate voltages. (You can neglect lattice vibrations and electron-electron interactions.) Give the expected form of $C(T)$ *versus* T for several Fermi energies, such as -0.2 eV, -0.1 eV, +0.1eV, and +0.2 eV.

Nuclear Physics Breadth

a) (5 points) Two nucleons have only one bound state, the deuteron d. It has isospin $I = 0$ and total angular momentum $J = 1$. What are values of its spin S and orbital angular momentum L allowed by Fermi statistics? What are the parities of those states? What would be the allowed S and L values if $I = 1$?

b) (5 points) The interaction can be approximated by a spherical potential well with depth $-V$ and radius a. Using $a = 1$ fm and approximating the wave function inside the well by a constant, estimate what fraction of the time the particles spend outside the well, $r > a$. The binding energy of d is $B = -E = 2.2$ MeV.

c) (5 points) Write down the Schrödinger equation for this problem, and the condition following from continuity of the wave function at $r = a$. Assuming that the $I = 1$ state has zero binding (in reality, it has a virtual state with very small positive energy) while $I = 0$, the deuteron, has B=2.2 MeV, find the depth of the potential V for both cases and compare them.

d) (5 points) If the $I = 1$ state would happen to be bound (in some alternative universe) how would it affect the big-bang nucleosynthesis and the life of stars?

High Energy Breadth

Three particles thought to be well described as $c\bar{c}$ quark anti-quark states are the spin-1 negative parity J/ψ, the ψ', and the ψ'', with masses of 3097, 3686, and 3771 MeV, respectively. Compare the spacings of these masses with what you would get from a hydrogenic formula, and give at least one reason to explain the discrepancy. As backkground, the lowest mass particle carrying charm, the D meson, has a mass of approximately 1869 MeV.

Astronomy Breadth 1

The classical Jean's criterion for gravitational instability is that disturbances of mass exceeding the Jean's mass are gravitationally unstable and subject to collapse. To within factors of order unity, the Jean's mass is given by

$$M_J = \left(\frac{kT}{G}\right)^{3/2} n^{-1/2} m^{-2},$$

where k is the Boltzmann constant, T is the temperature of the medium, G is the gravitational constant, n is the particle density of the medium, and m is the mean mass of the particles comprising the medium.

a) (7 points) This estimate of the Jean's mass may be derived as follows: A matter perturbation of characteristic size L and characteristic mass $M = nmL^3$ will collapse gravitationally if its internal pressure $P = nkT$ cannot withstand the pressure exerted by the weight of a column of the medium of length L. Derive equation (1) by comparing the internal pressure and the pressure exerted by the weight of a column of the medium of length L.

b) (7 points) Assume that the matter temperature matched the radiation temperature at the epoch of recombination and show that at the recombination epoch at redshift $z \approx 1000$ (again to within factors of order unity) the Jean's mass is given by

$$M_J = \left(\frac{kT_0}{G}\right)^{3/2} n_0^{-1/2} m^{-2},$$

where T_0 is the present-day radiation temperature and n_0 is the present-day particle density of the medium.

c) (6 points) Consider the matter content of the Universe to consist entirely of protons, and take the baryon-to-photon ratio to be $\eta = 6.1 \times 10^{-10}$ (as measured by WMAP), the present-day particle density of microwave background photons to be $n_0(\text{photon}) = 422\ \text{cm}^{-3}$, and the present-day radiation temperature to be $T_0 = 2.7$ K. Use equation (2) to estimate the Jean's mass at the epoch of recombination. How does the result compare to the mass of a globular star cluster, of mass $M \sim 10^5\ M_\odot$? (Note: The Boltzmann constant is $k = 1.4 \times 10^{-16}$ erg K^{-1}, the gravitational constant is $G = 6.7 \times 10^{-8}$ cm^3 g^{-1} s^{-2}, the proton mass is $m = 1.7 \times 10^{-24}$ g, and the mass of the Sun is $M_\odot = 2.0 \times 10^{33}$ g).

Astronomy Breadth 2

In about 1948, the Russian astronomer V.A. Ambartsumian hypothesized that the OB associations were regions of recent star formation. He based this on two observations:

a. Main sequence O and B stars are the most massive stars in OB associations, and

b. The average density of OB associations is too small for them to be gravitationally bound against galactic tides.

Justify Ambartsumian's hypothesis.

a) (8 points) What is the main sequence lifetime of a 20 solar mass B0 star? You should consider the following:

- The energy source in main sequence stars is H to He fusion.
- The atomic mass of ^{1}H is 1.6726×10^{-27}kg.
- The atomic mass of ^{4} He is 6.690×10^{-27}kg.
- The solar luminosity is 4×10^{33}erg/s.
- About 10% of the mass of star is in its core; nuclear fusion occurs only in the stellar core.
- The mass of the Sun is 2×10^{33} gm
- Luminosity of main sequence stars is roughly proportional to the cube of their mass.

b) (6 points) The typical velocity dispersion of an OB association in one dimension is 2 km/s. Is an association of mass M=10^4 solar masses, with a typical radius R of 10 parsec, gravitationally bound? (One parsec = 3.1×10^{16} meters.)

c) (6 points) The solar neighborhood is at D=8000 pc from the galactic center. Model the galactic potential as a 10^{10} solar mass point source (M_{gal}) at the galactic center. Determine if the 10^4 solar mass, 10 pc association is stable against galactic tides.

Remember that you must do at least one Breadth, and at least one Experiment question.

Atomic, Molecular, and Optical, Experiment

Moseley's law, which relates the energies of emitted K_α x-rays from an atom to atomic number, Z, can be studied by x-ray fluorescence.

a) (4 points) Write down Moseley's law. Define all symbols and explain why the energy scales with atomic number the way it does.

b) (4 points) Give the electronic configuration (i.e. which orbitals have how many electrons) for copper (atomic number 29). Indicate which levels are involved in the emission of K_β and L_α x-rays. Why is the K_α line actually split into $K_{\alpha 1}$ and $K_{\alpha 2}$ components?

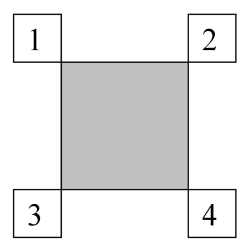
c) (4 points) Discuss how a copper atom can emit a K_α x-ray, including how the atom could be excited.

d) (4 points) Given that the Cu K_α energy is 8.05 KeV, estimate the K_α energy for Mo (atomic number 42).

e) (4 points) Most atomic physics experiments are carried out in gas phase at low density, and many involve cold samples. Why can a Moseley's law experiment be carried out with a bulk sample at STP (Standard Temperature and Pressure, 300 K, 1 Atm)?

Condensed Matter Experiment

A metallic plate of conductivity σ and thickness d is cut to a square shape ($d << a$, where a is the side of the square). Contacts are placed on the four corners.



a) (5 points) A constant current source of current I is connected to contacts 1 and 2, and the voltage is measured between contacts 3 and 4. How will the voltage depend on the thickness d and on the linear size a? Explain your answer in terms of simple physical arguments.

b) (5 points) The current source is connected to contacts 1 and 4, and the voltage is measured between 2 and 3. A magnetic field of magnitude H is applied perpendicular to the plate. Assuming ideal geometry as shown, how does the voltage scale with d, a and H?

c) (5 points) The plate is placed between a source and a detector of microwaves. The incident (P_i) and transmitted (P_t) power are measured, and the microwave transmission coefficient T is defined as $T = P_t/P_i$. For $d << \lambda$, where λ is the wavelength of the radiation, it is found that $T = (1+\sigma Q)^{-2}$, where Q depends on the thickness and universal constants (permeability of vacuum, μ_0 = 12.56 x 10^{-7} Vs/Am, permittivity of vacuum, ε_0 = 8.845 x 10^{-12} As/Vm). Up to a numerical factor, what is Q as a function of d, μ_0, and ε_0?

d) (5 points) If we want to make films of bismuth ($\sigma \sim 1 \times 10^6\ \Omega^{-1}\text{m}^{-1}$) with a microwave transmission coefficient of approximately 50%, what should be the thickness of the film? (Order of magnitude is sufficient for the answer.)

Nuclear Physics Experiment

This problem addresses momentum measurements and particle identification of charged particles like π and p frequently observed in Au-Au collisions at RHIC.

a) (6 points) A charged particle traversing a magnetic field is deflected by an angle $\Delta\phi$ according to

$$\Delta\phi(\text{mrad}) = \frac{0.3 \int B\, \text{d}L}{P},$$

where B is in Tesla, L is in meters, and P is in GeV/c. Assume two detectors measure the angle before and after the field with a precision of 1 mrad. Determine the relative momentum resolution for a 2 Tm field integral as a function of P. (Neglect multiple scattering.)

b) (4 points) Name at least two detector types used for charged particle tracking and explain in 2-4 sentences how they function and what they measure.

c) (6 points) Many different methods are used to identify particles, each useful in a particular momentum range. Study a system that measures the time of flight between the interaction point and a scintillator wall 5 meters away with a precision of 80 ps. Estimate up to what momentum can π (m_π = 140 MeV/c^2) and p (m_p = 938 Mev/c^2) be distinguished at the 3σ level?

d) (4 points) Name at least two other detector types used for particle identification. Give a 2-3 sentence or key work explanation how they function and what momentum range or particles they are used for.

High Energy Experiment

The scattering of high energy muons from nucleons can provide information about the internal structure of the protons and neutrons (Deep inelastic scattering or DIS). A scattering event is described in terms of q^2, the square of the four-momentum transferred to the target, and ν, the energy lost by the muon in collision. In the quark model, the interaction is viewed as an elastic collision between a muon and a stationary quark, where the quark carries a faction x of the nucleon's momentum.

a) (4 points) Draw a leading order Feynman diagram for this process and show the beam fragments and target fragments.

b) (4 points) Draw a schematic diagram of a fixed target experimental setup for deep inelastic muon scattering and comment on which components of the apparatus would be needed for:

a. Deep inelastic inclusive muon scattering experiment.

b. Deep inelastic semi-inclusive muon scattering experiment.

c. Deep inelastic exclusive muon scattering experiment. What would be the difficulties for such a measurement for a fixed target experiment? (Comment on one or two main difficulties only).

c) (4 points) Calculate x in terms of q^2, ν and the nucleon mass.

d) (4 points) Using simple arguments, calculate the ratio of the cross sections for muon-proton scattering and muon-deuteron scattering, assuming $q^2 \approx 100\ (\mathrm{GeV/c})^2$.

e) (4 points) Measurements of nucleon structure indicate that only half of the momentum of a fast moving nucleon is carried by objects which scatter muons. What carries the rest of the nucleon momentum, and why is it transparent to muons?

Astronomy Experiment 1.

RXJ185635-3754 was detected serendipitously during routine x-ray satellite observations a decade ago. It was quickly realized that the object was likely a nearby (now estimated at 120 pc), isolated (ie, we were seeing directly the thermal emission from its surface) neutron star in the late stages of cooling down after formation. The star is extremely faint in the visible, with a radius estimated at around 10 km and surface temperature in the range of a few $\times 100,000$ K. However, the object is reasonably bright in x-rays and this has raised interest in the possibility of measuring its diameter using a Michelson interferometer operating in the soft x-ray spectral region (~ 50 Å).

a) (5 points) What would be the baseline required to make such a measurement? Be careful to define what quantity an interferometer measures and what feature of that measurement you'd use to determine the diameter ($1\,\mathrm{pc} \sim 3.1 \times 10^{16}\,\mathrm{m}$).

b) (5 points) Assuming a surface temperature of 30 ev, collecting areas of 1 m^2 for each aperture, and a bandpass of 1 Å, how much time would it take to measure a diameter accurate to 10%? Assume that a diameter accurate to 10% is equivalent to making three measurements at suitable baselines, each accumulating 1000 photons, that the interferometer/detector system is 100% efficient and that there is no intervening absorption ($B_\lambda d\lambda = \frac{2\pi hc^2}{\lambda^5}(\exp\{hc/\lambda kT\} - 1)^{-1}$).

c) (5 points) RXJ185635-3754 is actually thought to have a hot spot that produces most of the flux at 50 Å. Assuming a uniform spot of 2 km radius with a temperature of 60 ev, what fraction of the flux received is from the spot? If you wanted to measure the spot's size directly, how long would the baseline have to be, how long would it take?

d) (5 points) The cooler surface of the star rapidly dominates the flux as one goes to longer wavelengths. Assuming that, what would be the best wavelength to measure the diameter of the star itself, at least in terms of the emitted flux? Why do you think they aren't planning to make the measurement in that wavelength region?

Astronomy Experiment 2

Treat the planned James Webb Space Telescope (JWST) telescope as a single circular lens with an effective focal length of $F = fD$, where f is the f-ratio of the final image falling on a detector, and D the diameter of the lens (or mirror), in meters. Being a space telescope, JWST is capable of near-perfect imaging, and we can ignore the effects of atmospheric aberrations. The NIRCAM detector is an infrared focal plane array with square pixels that are 18.5μm on a side. The full-well of each pixel is 10^5 electrons, the detector contains 2048×2048 pixels, and the quantum efficiency of the telescope-detector combination is 0.5. For numerics, 2×10^5 seconds of arc equals one radian.

a) (2 points) Express JWST's diffraction limit in seconds of arc at a wavelength $\lambda_{\mu m}$, where $\lambda_{\mu m}$ is the wavelength expressed in microns.

b) (4 points) Suppose you want two pixels across a resolution element (*i.e.*, Nyquist sampling) at a wavelength of 2μm, and that $D = 6.5$m. What is the required f-ratio of the final NIRCAM image? Does this answer depend on D?

c) (2 points) How many arcseconds across (on sky) is each pixel, and what is the field of view across a side of the detector?

d) (6 points) At a given wavelength of monochromatic light from a distant star of a given brightness, how does the integrated energy in an unaberrated stellar image (the point-spread function, or PSF) depend on D (ignore optical aberrations and atmospheric effects)? How does the peak intensity of a perfect PSF depend on D?

e) (6 points) Given that a zero magnitude star emits 1.5×10^9 photons m^{-2} s^{-1} in the astronomical K band (whose central wavelength is about $2\mu m$), and the shortest exposure time is 2 seconds, what is the magnitude of the brightest star that will just saturate the brightest pixel of its image? Assume that 13% of the point-spread function's energy falls on the brightest pixel, and that each detected photon releases one photo-electron into the capacitive well of a pixel. A factor of ten equals 2.5 stellar magnitudes.

Remember that you must do at least one Breadth, and at least one Experiment question. Turn in only four solutions.