

STONY BROOK UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

Comprehensive Examination, January 31, 2008

General Instructions: Twelve problems are given; you should do any four. If you do more than four problems, you must choose which four should be graded, and only submit those four.

Each problem counts 20 points, and the solution should typically take less than 45 minutes.

Use one exam book for each problem, and label it carefully with the problem topic and number and your name.

You may use a one page help sheet, a calculator, and with the proctor's approval, a foreign language dictionary. No other materials may be used.

Astronomy 1

It appears that the Universe is very close to spatially flat with a density parameter $\Omega \approx 1$ made up of a present-day matter density parameter $\Omega_{m_0} \approx 0.3$ and a present-day dark energy density parameter $\Omega_{\Lambda_0} \approx 0.7$. Take the dark energy equation of state to be the vacuum-energy-density equation of state

$$p_{\Lambda} = -\epsilon_{\Lambda}$$

relating pressure p_{Λ} and energy density ϵ_{Λ} , i.e., assume that the dark energy is equivalent to a cosmological constant.

a) (5 points) Write down the matter density $\rho_m = \rho_m(R)$ and the dark energy density $\rho_{\Lambda} = \rho_{\Lambda}(R)$ as functions of the scale factor R .

b) (5 points) Starting from the Friedmann equation for a spatially-flat Universe,

$$\dot{R}^2 = \frac{8\pi G \rho R^2}{3},$$

where G is the gravitation constant, and ρ is the total mass-energy density (in mass density units), show that the ultimate fate of a Universe with $\Omega_{m_0} = 0.3$ and $\Omega_{\Lambda_0} = 0.7$ is exponential expansion with time scale $H_0 \Omega_{\Lambda_0}^{1/2}$, where $H_0 = (\dot{R}/R)_0$ is the present-day Hubble constant.

c) (5 points) Now consider a spatially-flat Universe with a *negative* dark energy density, i.e. with $\rho_{\Lambda_0} < 0$. A negative dark energy density is equivalent to a negative cosmological constant, which produces a cosmic attraction. Show that the expansion of such a Universe comes to a halt when the scale factor reaches $R_{\max}/R_0 = (-\Omega_{m_0}/\Omega_{\Lambda_0})^{1/3}$.

d) (5 points) After the expansion of such a Universe comes to a halt, the Universe contracts. Show how to calculate the age of the Universe when the scale factor contracts to $R = 0$, expressing the answer in terms of an integral, i.e., write down the integral but do not evaluate it.

Astronomy 2

Consider a spherical HII region ionized by a star that radiates 10^{48} photons/sec shortward of 912\AA . The electron and proton densities are both equal to 10^4 cm^{-3} , the temperature of the plasma is 8000 K, and the effective recombination coefficient is $\alpha^{[2]} = 3 \times 10^{-15} \text{ sec}^{-1}$.

- a) (6 points) Consider a specific proton. How long before it recombines with an electron?
- b) (6 points) Once the electron and proton in (a) have recombined, roughly how long before the electron cascades to the ground state?
- c) (8 points) Assuming ionization equilibrium, what is the radius of the H II region?

Astronomy 3

Type Ia supernovae are used as standard candles because they are bright (hence visible at great distances), and because they are thought to reach a uniform peak brightness. (Actually they don't, but it is thought that the peak brightness correlates with the decay time, so it can be calibrated.) The typical type Ia supernova reaches an absolute magnitude of -19.3 ± 0.3 .

- a) (4 points) What is a type Ia supernova? What explodes? What is likely to be left behind?
- b) (4 points) The absolute magnitude is the magnitude that an object will have if seen from a distance of 10 parsecs (32.6 light years). What is the apparent brightness of a Type Ia supernova at a distance of 100 Mpc?
- c) (4 points) If the supernova occurs at a low galactic latitude, there may be significant interstellar extinction. Assume a standard reddening law. You measure the B-V color of an A0 star in this direction to be 1.0. What is the likely value of the visual extinction in magnitudes?

You plan to image the supernova to determine its brightness in the V band. A source with $V=0$ yields $\lambda F_{\lambda} = 1.98\text{E-}5 \text{ erg/cm}^2/\text{s}$. The V band is centered at 550 nm, and has a width of 100 nm. You may assume the filter transmission is flat; that is, it is 1 from 500 to 600 nm and 0 elsewhere. You are using the CTIO 1.3m reflector. The ANDICAM images has a system efficiency of 25% in the V band (which includes the area blocked by the secondary mirror). The sky brightness is 1 count per pixel per second. The plate scale is 0.4 arcsec/pixel. Assume the seeing is 10.0 arcsec, and you measure the source counts in a 7×7 pixel square aperture.

- d) (4 points) What is the count rate from the supernova?
- e) (4 points) How long do you have to integrate to measure the source flux to 1% accuracy? How long would you have to integrate in the absence of background?

Astronomy 4

This question considers the effect of the Malmquist bias on the determination of the Hubble law from the magnitude-redshift relation. The Malmquist bias occurs because, in a magnitude-limited sample, only more luminous galaxies are included in the sample at larger distances. The mean absolute magnitude of the sample galaxies is therefore distance-dependent, and the galaxies are not “standard candles”.

a) (5 points) A reasonable approximation to the luminosity function of galaxies is

$$\Phi(M) = \begin{cases} C & M > M_* \\ 0 & M < M_* \end{cases}$$

Here the luminosity function measures the galaxy density per *magnitude* (unlike the common definition per luminosity interval), M_* is the absolute-magnitude cutoff, and C is a constant. Show that if the sample is complete, or if the galaxies are chosen at random from a complete sample, then the mean absolute magnitude of the galaxies at distance r is

$$\langle M \rangle = \frac{1}{2} \left(m_L - 5 \log \frac{r}{10 \text{ pc}} + M_* \right),$$

where m_L is the limiting magnitude of the sample, and r is sufficiently small, so that galaxies at the cutoff absolute magnitude M_* are bright enough to be included in the sample.

b) (5 points) Deduce that the mean apparent magnitude at distance r for galaxies in this sample is

$$m = 2.5 \log r + \text{const.}$$

c) (10 points) A regression of the observed m on $\log v$, where $v \ll c$ is the recession velocity, yields

$$m = 2.5 \log v + \text{const.}$$

Show that one can therefore deduce the linear Hubble law

$$v \propto r \quad ,$$

provided that the Malmquist bias is taken into account. Also show that, if the Malmquist bias is ignored, and the galaxies are taken to be “standard candles”, then the Hubble law is erroneously deduced to be quadratic

$$v \propto r^2 \quad .$$

(Such a relation is, in fact, expected in a closed universe at the point of maximum expansion. Failure to account for the Malmquist bias might therefore lead one to believe that the universe is indeed closed and at the point of maximum expansion.)

Atomic, Molecular, and Optics 1

The Bohr formula for the energies of the states of atomic hydrogen (H) are given by $E_n = -mc^2\alpha^2/2n^2$, where m is the electron rest mass, $\alpha = e^2/\hbar c \approx 1/137 \approx 7.3 \times 10^{-3}$ is the Sommerfeld fine structure constant, and n is an integer ≥ 1 .

(a) (5 points) The Bohr formula can be readily derived from the classical force on an electron in the Coulomb field of the proton, and one additional condition. State that additional condition, and use it to derive the result.

(b) (5 points) The advent of the Schrödinger equation to describe H led to the same energy levels. When this is done in (three-dimensional) spherical coordinates, two new quantum numbers are introduced in addition to the n of the Bohr energy formula. What are these, and what is their meaning?

(c) (5 points) Energy levels which are degenerate in solutions to the Schrödinger equation are split by an interaction known as fine structure. What is the physical origin of these shifts of the energy levels of H? How do they arise? Sketch how they might be estimated. (Outline the steps without following through the calculation).

(d) (5 points) Draw three energy level diagrams for the lowest energy states of H ($n \leq 3$). For the first one, show only the Bohr levels and give the energies numerically in some units that you find convenient. For the second one, show the consequences of the two new quantum numbers that arise from the solutions of the Schrödinger equation. For the third one, include the fine structure. Be sure to give the approximate magnitudes of all the energy intervals and their spacings, since drawing to scale would not be able to show this. Describe how these states are normally labeled, and give the standard notation for at least one of the states.

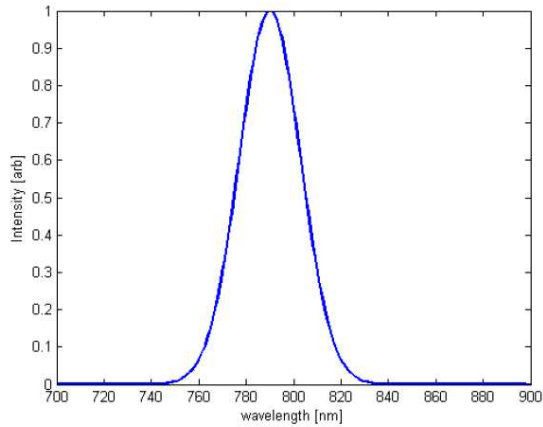
Atomic, Molecular, and Optics 2

An optical spectrometer makes use of a grating to diffract light of different colors to different angles and therefore different positions on a detector plane some distance away. Consider a spectrometer composed of a slit, two curved mirrors (both of focal length f), a diffraction grating consisting of N grooves separated by a distance h , and a spatially sensitive detector (e.g., photographic film or a ccd camera). Such a configuration is known as a Czerny Turner spectrometer.

(a) (5 points) Draw a picture (block diagram) of what such a spectrometer should look like using the components listed. Explain your placement of these items and label distances wherever they are fundamental to the operation of the spectrometer. Why is there a slit and how does it affect the spectral resolution of the spectrometer?

(b) (10 points) Now assume that the slit is narrow enough and the spatial resolution of the detector is fine enough that that neither contributes to the resolution. For perfectly monochromatic light, what is the angular width of the n^{th} order diffracted beam in terms of n and the total width Nh of the grating? Use this to express the resolving power $\lambda/\Delta\lambda$ of the spectrometer in terms of the parameters given. (For simplicity, assume that the light falls on the diffraction grating at normal incidence.) What is the resolving power of a 50mm grating with 2000 grooves/mm using the first diffraction order?

(c) (5 points) Imagine that you made use of the spectrometer you designed above to measure the spectrum (intensity vs wavelength) of a short laser pulse and you measured the spectrum shown below. Estimate the shortest pulse (FWHM of $I(t)$) that could have produced the spectrum you measured.



High Energy Physics 1

In a typical hadron collider experiment (such as ATLAS, CMS, D0 or CDF), describe the method used to select and (as closely as possible) measure the energy of any five the following particles (4 points credit each):

- a) an electron
- b) a photon
- c) a muon
- d) a tau lepton
- e) a light quark
- f) a b-quark
- g) a neutrino
- h) a SUSY LSP (lightest supersymmetric particle, undetected to date)

High Energy Physics 2

The next decade will clarify some fundamental problems in particle physics.

- a) (5 points) Which particles of the Standard Model of particle physics get a mass from the Higgs effect? Which interaction gives a mass to the electron? Why is it not allowed in the Standard Model to add an explicit mass term for the electron by hand (i.e., not using the Higgs mechanism)? State a lower bound on the Higgs mass.
- b) (5 points) If neutrinos have a nonzero mass, they have right-handed components. What are their $SU(3) \times SU(2) \times U(1)$ quantum numbers?
- c) (10 points) Derive the probability $P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{(E_1 - E_2)}{2\hbar} t$ for the oscillation of a muon-neutrino into a tau-neutrino. Define all symbols in this equation.

Condensed Matter 1

This problem illustrates some physics of the integer quantum Hall effect. Consider an electron gas confined to the two dimensional xy plane, in a perpendicular magnetic field B . Let us neglect the interaction between electrons. The Hamiltonian of a single particle is given by

$$H = \frac{1}{2m} \left(-i\hbar \vec{\nabla} + \frac{e}{c} \vec{A} \right)^2 + V(x, y),$$

where \vec{A} is the vector potential of magnetic field and $V(x, y)$ is an additional electrostatic (confining) potential. \vec{A} can be taken in the Landau gauge, $A_x = -By$, $A_y = 0$. For simplicity we will take the confining potential to be a one-dimensional harmonic potential $V = \frac{1}{2}m\omega_0^2 y^2$.

- (a) (5 points) Separate variables as $\psi(x, y) = \psi_k(y)e^{ikx}$ and write the stationary Schrodinger equation for $\psi_k(y)$. Cast this equation into the form of a one-dimensional harmonic oscillator, and find the energy levels $E_{k,n}$ with $n = 0, 1, 2, \dots$. The levels at given n are said to belong to the same *Landau level*.
- (b) (5 points) Let us assume that the chemical potential μ is such that Landau levels with $n > 0$ are empty (i.e., $E_{k,n} > \mu$ for $n > 0$). Then the only occupied states are the ones with $n = 0$. What are the maximum and minimum values of k for the occupied levels?
- (c) (5 points) What are the positions (in the y direction) of those occupied levels?
- (d) (5 points) The states with maximal and minimal k are called the edge states of the integer quantum Hall effect. Find their velocity.

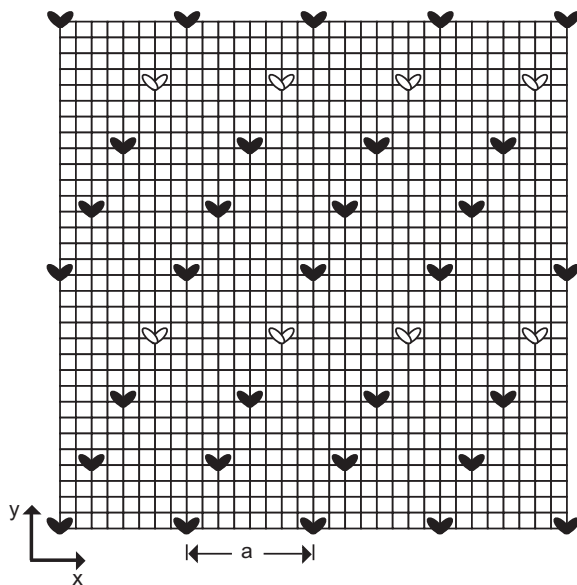
Condensed Matter Physics 2

An arrangement of black and white flies, as shown in the figure below, are found by a graduate student on the clean-room wall.

(a) (7 points) Assuming the pattern goes on forever, find a set of primitive lattice vectors and basis vectors to describe this two-dimensional fly crystal. Find an expression for the reciprocal lattice vectors and sketch the reciprocal lattice.

(b) (7 points) The student decides to obtain a diffraction pattern for the fly structure to show her advisor. She uses electromagnetic radiation incident along the x-direction and wants to obtain a diffraction spot in the x-y plane but not along the x-axis. (Assume that black and white flies have different amplitudes for scattering the radiation.) Find the minimum incident wave vector that she needs to get at least one such diffraction spot? If $a = 10\text{cm}$, what type of radiation (x-ray, UV, visible, etc) does she need?

(c) (6 points) The student manages to observe the diffraction spot corresponding to the $\mathbf{K} = \frac{2\pi}{a}\hat{x} + \frac{\pi}{a}\hat{y}$ reciprocal lattice vector. But soon after, the radiation starts frying the flies, so that all the flies can now be considered black. What happens to the intensity of the spot that the student was monitoring? (Give a quantitative reason.)



Nuclear Physics 1

The nuclear Hamiltonian is given by

$$H = H_{\text{strong}} + H_{\text{electromagnetic}} + H_{\text{weak}}$$

The aim of this problem is to show that CP violation of the nuclear Hamiltonian can in principle be observed from the distribution of the spacing of its eigenvalues. Each section is worth 4 points.

(a) Which of the three parts breaks parity P and which PC (with C charge conjugation)?

(b) We can also write the Hamiltonian as

$$H = H_0 + H_P + H_{PC},$$

where H_0 is invariant under P and PC , H_P is invariant under PC but not P and H_{PC} breaks both P and PC . If $|\alpha+\rangle$ and $|\alpha-\rangle$ are two states of opposite parity show that

$$\langle \alpha+ | H_0 | \alpha- \rangle = 0.$$

(c) It can be shown that as a consequence of PC invariance (or T (time reversal)), the matrix elements of the Hamiltonian can be chosen real. Now assume that for two states, the PC invariant part of the nuclear Hamiltonian can be approximated by the 2×2 matrix.

$$H = \begin{pmatrix} a & c \\ c & -a \end{pmatrix}.$$

For a Gaussian distribution of a and c (both are real) obtain the small S behavior of the distribution of the spacing S between the eigenvalues λ_1 and λ_2 of H .

(d) Now assume that PC is broken so that H has the structure

$$H = \begin{pmatrix} a & c \\ c^* & -a \end{pmatrix},$$

and again assume that a, c have a Gaussian probability distribution $P(a, c, c^*) = \exp(-a^2 - cc^*)/\pi^{3/2}$. Find the small S behavior of the spacing distribution of the eigenvalues.

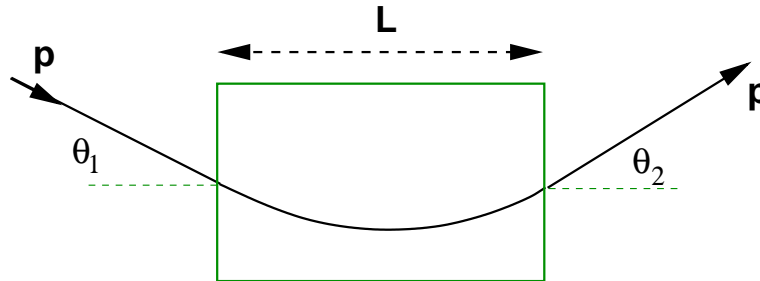
(e) Indicate how the difference in small S behavior of the spacing distribution can be used to obtain an upper limit for the PC breaking part of the nuclear Hamiltonian.

Nuclear Physics 2

We will consider and compare the measurement of the momentum of high energy charged particles in a magnetic spectrometer with the measurement of their energy in a calorimeter.

a) (5 points) Consider a charged particle of momentum p passing through a region of uniform magnetic field B oriented perpendicular to the page. Assume that the particle trajectory is confined to the plane of the paper, and that the magnetic field drops abruptly to zero outside of the region indicated. The particle enters the field region at angle θ_1 and leaves at an angle θ_2 , as shown in the figure. Derive an exact relation for θ_2 as a function of θ_1 , B , p , L , and the charge q .

NOTE: Use counter-clockwise as the positive sense for both angles, *i.e.*, the figure shows $\theta_1 < 0$ and $\theta_2 > 0$.



b) (5 points) If the entrance and exit angles are each measured with a resolution $\delta\theta$, derive the momentum resolution $\delta p / p$ as a function of momentum. Here you may assume that θ_1 and θ_2 are both small.

c) (5 points) List the dominant energy loss mechanisms for high energy ($1 \text{ GeV} < E - mc^2 < 100 \text{ GeV}$) electrons and muons in matter. (Give separate answers for the two if they are not the same.) Which has the larger range?

d) (5 points) A sampling calorimeter may be used to measure the energy of an incident electron, by converting a fraction of its energy to charged particles, and measuring the total energy deposited. What is the functional form of the energy resolution $\delta E / E$ of a sampling calorimeter? (No numerical factors required).