STONY BROOK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY

# COMPREHENSIVE EXAMINATION
September 3, 2008

**General Instructions:** Twelve problems are given; you should do any four. If you do more than four problems, you must choose which four should be graded, and only submit those four.

Each problem counts 20 points, and the solution should typically take on the order of 45 minutes.

Unless otherwise noted, each section of a problem is equally weighted. Please make sure that you have done all of the parts of each question you solve.

Use one exam book for each problem, and label it carefully with the problem topic and number and your name.

You may use a one page help sheet, a calculator, and with the proctor's approval, a foreign language dictionary. No other materials may be used.

Some potentially useful information:

- The atomic mass of hydrogen is 1.00794 amu.
- The atomic mass of helium is 4.002602 amu.
- 1 amu is $1.66 \times 10^{-27}$ kg.
- The speed of light is $c = 2.998 \times 10^8$ ms$^{-1}$.
- The gravitational constant is $G = 6.673 \times 10^{-11}$m$^3$kg$^{-1}$s$^{-2}$.
- The solar luminosity is $3.846 \times 10^{26}$ W.
- The mass of the Sun is $1.989 \times 10^{30}$ kg
- The radius of the Sun is $7.00 \times 10^8$ meters

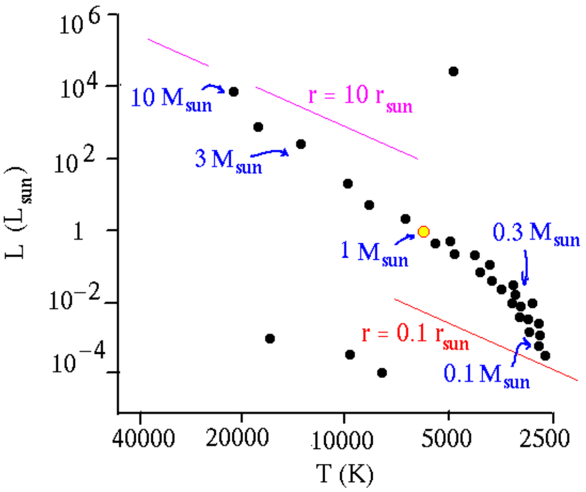
## Astronomy 1

Timescales are important in astrophysics in part because they give insight into (unseen) physical processes. Use the following back-of-the-envelope calculations to show that the Sun and the stars are powered by nuclear fusion.

a) (5 points) Suppose that the source of the solar energy were gravitational potential energy. What is the gravitational potential energy of the Sun (to constants of order unity)? How long does it take the Sun to radiate this amount of energy, and what would happen to the Sun after it radiates it?

b) (10 points) How long can nuclear fusion power the Sun? Assume that only 10% of the Solar mass in the core partakes in the reactions, and that neutrino losses are negligible. State your answer in years.

c) (5 points) Estimate the main-sequence lifetime (in years) of a 10-solar-mass star. You may find the following H-R diagram useful.



## Astronomy 2

Consider a hypothetical globular cluster with a gravitational potential

$$\Phi(r) = -\frac{GM}{(r^2+b^2)^{1/2}} \quad , \tag{1}$$

where $G$ is the gravitational constant, $M$ is the mass of the cluster, $b$ is its core radius, and $r$ is the distance from the center of the cluster. (This is the potential smoothed over a few interstellar separations; it does not include near-neighbor interactions.)

A. (10 points) Show that the cluster mass enclosed within radius $r$ is given by

$$M(r) = \frac{Mr^3}{(r^2+b^2)^{3/2}} \quad , \tag{2}$$

and deduce that the density of the cluster satisfies

$$\rho(r) \propto \left(r^2+b^2\right)^{-5/2} \quad . \tag{3}$$

B. (10 points) Assuming that the cluster is held up by isotropic pressure due to the random motions of the stars in the cluster, show that the pressure profile satisfies

$$P(r) \propto (r^2 + b^2)^{-3} \qquad , \tag{4}$$

and deduce that the cluster has a polytropic equation of state of the form

$$P = K\rho^{\gamma} \qquad , \tag{5}$$

with $\gamma = 1.2$.

## Astronomy 3

Consider the very early universe, for which the dynamics of the expansion is dominated by radiation (rather than by matter or by vaccuum energy) with an equation of state

$$p = \frac{1}{3}\rho c^2 \tag{6}$$

relating pressure $p$ and energy density $\rho c^2$ (where $\rho$ is the equivalent mass density and $c$ is the speed of light), i.e., consider a universe that contains *only* radiation.
The "flatness" or "oldness" problem of standard big-bang cosmology states that at early epochs the density parameter $\Omega = \rho/\rho_c$ (where $\rho_c$ is the critical density) must have been extremely close to $\Omega = 1$, otherwise the universe would not exist as we know it today.

A. (5 points) Write down an equation for the dependence of $\rho$ on the scale factor of the universe, $R$.

B. (10 points) Suppose that at time $t_1 = 10^{-40}$ s, the density parameter had the value $\Omega_1 = 0.9999$. Starting from the Friedmann equation

$$\dot{R}^2 = \frac{8\pi G\rho R^2}{3} + (1 - \Omega_1)H_1^2 R_1^2, \tag{7}$$

where $G$ is the gravitation constant and $H_1$ and $R_1$ are the Hubble parameter ($\dot{R}/R$), and scale factor at epoch 1, determine the scale factor $R_2$ (in units of $R_1$) at the epoch 2 at which the density parameter had dropped to the value $\Omega_2 = 0.0001$. Also, give an upper limit to the age of the universe at epoch 2.

C. (5 points) Briefly discuss some *observational evidence* that rules out the cosmological model in B.

## Astronomy 4

The interstellar medium contains both dust grains and gas, with the quantity of dust $\sim 1\%$ by mass. Suppose that the same dust were present in the Earth's atmosphere, in the same proportion by mass to the gas. Estimate how far a person on the Earth's surface would be able to see (visible light). Be sure to explain your reasoning; don't just quote an answer.

## Atomic, Molecular, and Optical Physics 1

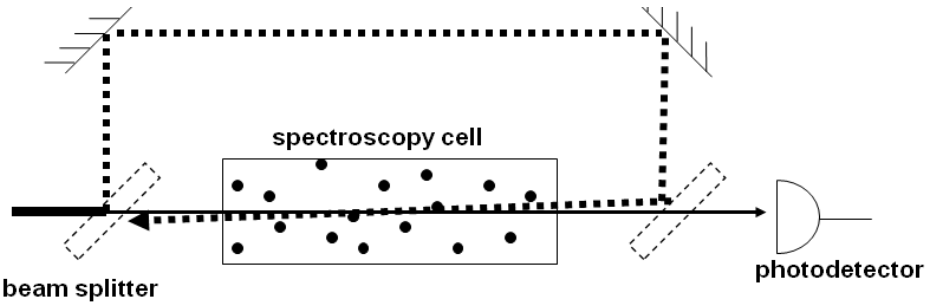
Laser spectroscopy of atomic transitions is an important tool for high-precision frequency stabilization of lasers in AMO experiments.

Consider a weak laser beam of variable frequency $\nu$ ("probe") that is sent through a spectroscopy cell containing an atomic gas, and subsequently hits a photodetector. The laser frequency is swept across an atomic resonance which occurs at frequency $\nu_0$. and the signal $S(\nu)$ is recorded with the detector, showing a transmission dip $S(\nu) \propto [1 - L(\nu)]$ centered around $\nu_0$, where $L(\nu)$ denotes the lineshape function.

a) (4 points) What is the physical effect that gives rise to the dip? Discuss qualitatively the mechanisms that can contribute to the width of the dip.

b) (8 points) Derive an expression for the full width at half maximum (FWHM) of $L(\nu)$ arising from Doppler broadening due to thermal motion (neglecting the contributions from other mechanisms). Calculate the FWHM for a room-temperature gas of rubidium-87 atoms ($D_2$ transition wavelength: 780nm), and compare it to the natural linewidth $\Gamma/2\pi = 6$ MHz.

c) (8 points) In a so-called "Doppler-free saturation spectroscopy" setup as shown in the figure below, the laser beam is split into two parts. The weaker probe beam propagates from left to right into the photodetector, while a stronger beam ("pump" - dashed line) is introduced which is propagating in the opposite direction, and spatially overlaps the probe beam.



With the pump beam present, sketch the recorded photodiode signal $S(\nu)$, and describe its salient features. How does this configuration affect Doppler broadening? What determines the width of the dispersive region around $\nu_0$ , and what is a typical number for a very dilute gas?

## Atomic, Molecular, and Optical Physics 2

Consider the interaction Hamiltonian for a two-level atom with induced electric dipole moment $\mu$ in an oscillating electric field $E$,

$$H = \frac{\hbar}{2}\left[\begin{array}{cc} \Delta & -\chi \\ -\chi & -\Delta \end{array}\right] \tag{8}$$

where $\Delta$ is the detuning of the applied field from resonance, and $\chi = \frac{\mu E}{\hbar}$ is the Rabi frequency.

a. (4 points) What is the Rabi frequency for an atom with a dipole moment of $1 \times 10^{-30}$ Cm exposed to a laser beam with an intensity of $1 \times 10^{6}$ W/cm$^2$?

b. (8 points) Calculate the adiabatic eigenvalues of this Hamiltonian, and draw a picture of these eigenvalues as a function of time for (a) an adiabatic intensity change (i.e., a Gaussian laser pulse) at finite detuning and (b) for an adiabatic frequency sweep (i.e., a continuous-wave laser with a frequency that varies linearly in time) from below resonance to above resonance at finite Rabi frequency.

c. (8 points) Calculate the AC Stark shift in the limit that $\Delta \gg \chi$, and in the limit that $\chi \gg \Delta$.

## High Energy Physics 1

Currently there is great interest in the search for Standard Model Higgs Boson at the Large Hadron Collider (LHC) which nears completion at CERN, Geneva, Switzerland. The LHC accelerates and collides two high-intensity counter-rotating proton beams of 7 TeV ($7 \times 10^{12}$ eV) each, giving a total energy of 14 TeV in the center-of-mass (= the Laboratory). In the Standard Model, the Higgs (mechanism) is employed to provide mass terms for the Weak Vector Bosons $W$ and $Z$ by its coupling to the gauge fields. At the same time, the Higgs provides mass for fermions.

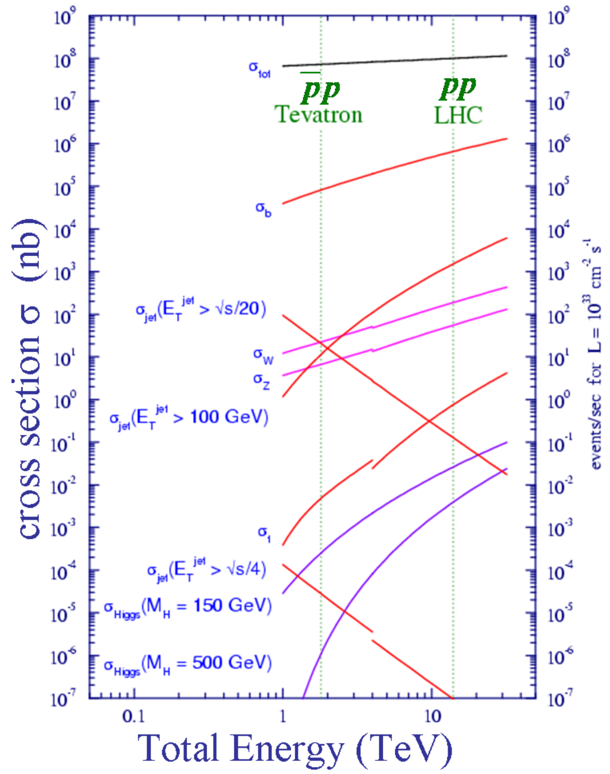
The LHC is designed to provide $10^{34}$ interactions per second per cm$^2$ of cross section per experiment. The cross section for the production of a Standard Model Higgs with 120 GeV mass, is about 30 pb (pico-barn; 1 barn equals $10^{-24}$ cm$^2$). For comparison, the total proton-proton cross section at 14 TeV is about 110 mb, and the cross section for production of jets with transverse momentum $p_T > 100$ GeV is about $1\mu$b; see the figures below and on the following page.

a) (4 points) Show that for a typical "LHC year" of running (30% efficiency), the integrated luminosity is about 100 events per fb (femto-barn) of cross section. Calculate the cross section and the number of events per LHC-year for the process $pp \to H + anything$, followed by $H \to \gamma\gamma$.

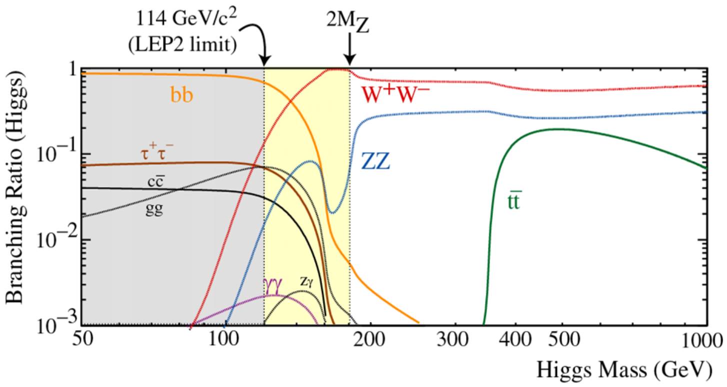
b) (4 points) The "golden" (most likely) discovery channel of the Higgs boson, if its mass is low ($m_H \simeq 130$ GeV), is in events in which the produced Higgs decays into two high-energy photons, $H \to \gamma\gamma$. Estimate the $x$-values ($x$ is the parton momentum fraction) that lead to Higgs <u>production</u> for $m_H = 120$ GeV.

c) (6 points) Explain, with the information provided to you, why the discovery of the SM Higgs in the $b\bar{b}$ decay channel is less likely than in the "golden" channel above.

d) (6 points) Provide a Feynman diagram depicting the dominant contribution(s) to the Higgs production cross section at low Higgs mass at the LHC.



Cross sections of important processes at high energies. The energies of the TeVatron Collider ($\bar{p}p$) and the LHC ($pp$) are indicated. The scale on the left runs from $10^{-7}$ to $10^{9}$ nanobarn.



Predicted Higgs decay branching fractions as function of the Higgs mass; as the Higgs mass increases, the decay fractions change dramatically. Note the experimental lower limit on the mass from the LEP2 experiments.

### High Energy Physics 2

a) (7 points) List the fermions of the first family of the Standard Model, together with their $SU(3) \times SU(2) \times U(1)$ quantum numbers. You may take for $U(1)$ either their electric charge or their hypercharge.

b) (7 points) Give 3 examples of superpartners of particles of the Standard Model. List their $SU(3) \times SU(2) \times U(1)$ quantum numbers and their spins.

c) (6 points) List three current experiments which test the Standard Model, or extensions of it. What aspects of the Standard Model or extensions of it do they test?

### Condensed Matter Physics 1

There is a vast array of experimental techniques used to characterize solids. For each of parts a. through d., give an explanation of the phenomenon and describe quantitatively what aspect of the sample it measures. Each answer should be about one paragraph; include a few equations if they help illustrate the concept.

a. (3 points) X-ray diffraction.

b. (3 points) Inelastic neutron scattering

c. (3 points) Hall effect

d. (3 points) Heat capacity of a metal at low ($< 1$K) temperature

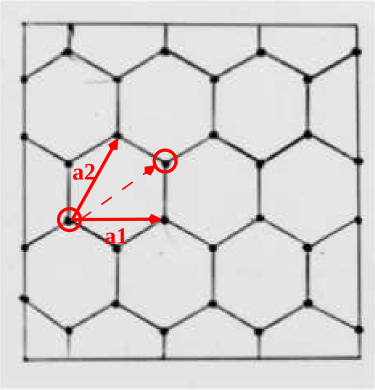
e. (4 points) Suppose you have a sample of material known to be an intrinsic semiconductor. You measure its resistance to be $6.25 \times 10^5 \Omega$ at $20^o$C, and $5400\Omega$

at 100$^o$C. If it is possible, estimate the band gap from the information given. If not, explain why not.

f. (4 points) Explain the difference between type I and type II superconductors, and draw a sketch of the magnetization vs. field that you would expect to measure from each.

**Condensed Matter Physics 2**

Graphene is a 2D crystal with carbon atoms arranged in a honeycomb structure. The lengths of the bonds are all equal and the angles between bonds are $2\pi/3$. The primitive lattice vectors and basis atoms are indicated on the figure.



(a) (4 points) Draw a labeled sketch of the first Brillouin zone.

Carbon nanotubes are rolled-up graphene sheets. Consider a single-walled carbon nanotube of radius $R$. As long as R is large enough (so that we can neglect curvature), the rolling-up of the graphene sheet can be thought of as imposing periodic boundary conditions on the graphene sheet in the circumferential direction (let us take that to be the $x$-direction). Because $R$ is finite (typically nanotubes with $R < 100\mathring{A}$ are seen experimentally), this gives rise to quantization of $k_x$.

(b) (4 points) Overlay on the Brillouin zone of the planar graphene sheet the lines of allowed k-vectors for the rolled-up sheet. For a nanotube of radius R, what is the perpendicular distance between adjacent lines?

Experimentally it has been found that at very low temperatures ($T < 30K$), the heat capacity of carbon nanotubes decreases linearly with T, even for semiconducting nanotubes where there are no free electrons. This seems surprising at first. After all, carbon nanotubes are just rolled-up graphene sheets; shouldn't they behave similarly?

(c) (6 points) Can you explain this behavior? [Hint: Define the very low-temperature limit as the regime where the thermal wave vector, $k_T$, satisfies $k_T \ll 1/R$. Draw on the same figure used in part (b) a circle of radius $k_T$ in this regime, as well as a circle corresponding to the opposite limit, $k_T \gg 1/R$.]

(d) (6 points) Using the Debye approximation, show that in the 1D limit the lattice heat capacity is proportional to T and follows this relation:

$$C_{ph} = \gamma \frac{3Lk_B^2 T}{\pi \hbar v_s} \quad \text{for} \quad T \ll \frac{\hbar v_s}{k_B R} \tag{9}$$

where L is the length of the tube, $v_s$ is the speed of sound, and $\gamma$ is a dimensionless number.

## Nuclear Physics 1

The level density for an infinite cubic well with side $L$ and potential zero at the bottom is given by

$$\rho(E) \sim L^3 \sqrt{E}. \tag{10}$$

On the other hand, according to the Bethe formula, the nuclear level density is given by

$$\rho(E) \sim \frac{1}{E^{5/4}} e^{b\sqrt{E}} \tag{11}$$

with $b$ a positive constant.

a) Give a two-line argument why there are exponentially more nuclear levels than you have levels in a cubic box.

Next, derive the leading exponent of the Bethe formula using a Fermi gas model of the atomic nucleus. The occupancy of the single particle states with energy $\epsilon_k$ is given by $m_k \in \{0, 1\}$ , such that $\sum_k m_k = m$ and $\sum_k \epsilon_k m_k = E_{\vec{m}}$. The level density is thus given by

$$\rho_A(E) = \sum_m \sum_{\vec{m}} \delta(A - m)\delta(E - E_{\vec{m}}). \tag{12}$$

It can be expressed in terms of the grand canonical partition function

$$Z(\alpha, \beta) = \sum_{m_k} e^{\alpha m - \beta E_{\vec{m}}} \tag{13}$$

as the inverse Laplace transform

$$\rho_A(E) = \frac{1}{(2\pi i)^2} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} d\alpha d\beta Z(\alpha, \beta) e^{-\alpha E + \beta E}. \tag{14}$$

b) If the single density of states is $g(\epsilon)$ show that

$$\log Z = \int_0^\infty g(\epsilon) d\epsilon \log(1 + e^{\alpha - \beta\epsilon}). \tag{15}$$

c) The low-temperature expansion of $Z$ is given by

$$\log Z = \alpha A - \beta E_0 + \frac{\pi^2}{6\beta} g(\epsilon_F) + \cdots \qquad \text{with} \qquad \epsilon_{\mathrm{F}} = \frac{\alpha}{\beta}. \tag{16}$$

Show that $A$ is the number of particles of the ground state and $E_0$ the energy of the ground state.

d) Using a saddle point approximation for the $\alpha$ and $\beta$ integrals, derive the leading exponent of the Bethe formula.

## Nuclear Physics 2

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory is the world's first and only heavy ion collider designed to study the high density partonic medium expected to be produced in nuclear collisions at high energy. The partonic interactions occurring in RHIC collisions produce an abundance of particles including photons ($\gamma$), electrons, muons, neutral and charged pions ($\pi^0, \pi^\pm$), neutral and charged kaons ($K^0, K^\pm$), protons ($p$), anti-protons($\overline{p}$), and deuterons ($d$).

The PHENIX detector measures transverse momentum ($P_T$) of the particles created in the RHIC collisions in a detector employing a magnetic field $B$ and a set of wire chambers to track the particle's trajectory. One of the ways the particle is identified is by employing a Time of Flight detector (TOF), which is based on the principle of recording the time taken for a particle to reach this detector measured from the instant of beam (bunch) collision. After the TOF detector is a layer of electromagnetic calorimeters.

a) (10 points) Based on the following observations made in the PHENIX experiment, conclude on the **most likely identity** of the particle. Note that each of the particle codes could correspond to more than one type of particles. Suggest at least one correct scenario.

| Particle code | hits in wire chambers | $P_T$ measured using $B$ | time of flight signal | EM Calorimetry energy deposited = $E$ | $E/P_T$ |
|---|---|---|---|---|---|
| A | yes | no | no, speed $v = c$ | yes | $\infty$ |
| B | yes | yes, 5 GeV/c | yes, slower than A | yes | $\approx 1$ |
| C | yes | yes, 5 GeV/c | yes, slower than B | yes | $< 1$ |
| D | yes | yes, 5 GeV/c | yes, slower than C | yes | $< 1$ |
| E | yes | yes, 5 GeV/c | yes, slower than A | no | $\approx 0$ |

One of the most exciting results from RHIC based on Au-Au collisions and their comparison with p-p collisions at center of mass energy of 200 GeV is shown in the figure below. The 0-10% indicates that the two colliding Au nuclei overlapped more than 90% geometrically i.e. more than 90% of the nucleons in each nuclei collided. One defines the nuclear modification factor $\mathrm{R}_{AA}$, to be the ratio of cross sections in Au-Au *vs.* p-p collisions, normalized to the number of nucleons in the colliding Au nuclei. The figure shows the $R_{AA}$ for neutral pions (black) and eta (red) particles created in Au-Au collisions to those produced in p-p collisions. Also shown with blue points are data which represent photons produced in such collisions (created directly in the nuclear collisions, and **not** those photons which might have originated in the decays of mesons created in the nuclear collisions).

b) (10 points) Provide an explanation of what you understand is the main physics message in this plot.