

STONY BROOK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY
Comprehensive Examination, January 28, 2009

General Instructions: Twelve problems are given; you should do any four. If you do more than four problems, you must choose which four should be graded, and only submit those four.

Each problem counts 20 points, and the solution should typically take less than 45 minutes.

Use one exam book for each problem, and label it carefully with the problem topic and number and your name.

You may use a one page help sheet, a calculator, and with the proctor's approval, a foreign language dictionary. No other materials may be used.

Elementary Particle Physics 1

There is ongoing interest in the phenomenon of particle to antiparticle oscillations, particularly in the B meson system and in neutrinos. This problem concerns oscillations in B mesons, a topic related to this year's Nobel Prize to Kobayashi and Maskawa.

- (a) (5 points) What meson properties are needed to allow oscillations?
- (b) (5 points) Draw the Feynman diagram(s) for $B^0 \rightarrow \bar{B}^0$ oscillations. How do CKM matrix elements enter?
- (c) (5 points) The quark (and thus meson) weak eigenstates differ from the mass eigenstates. The weak Hamiltonian (H) written in the strong eigenstate basis is given by

$$H \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \begin{bmatrix} (M_1 + M_2)/2 & (M_1 - M_2)/2 \\ (M_1 - M_2)/2 & (M_1 + M_2)/2 \end{bmatrix} \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}$$

with M_1 and M_2 the masses of the weak eigenstates and (used below) $\Delta M \equiv |M_2 - M_1|$. Show that this describes oscillations of the mass eigenstates by deriving the probability of finding strong eigenstate B^0 as a function of time, $P_{B^0}(t)$, assuming that $P_{B^0}(0) = 1$.

- (d) (5 points) How would the oscillation frequency be measured? Assume the B meson lifetime is ≈ 2 ps, the decay chain is $B^0 \rightarrow \pi^+ D^- \rightarrow \pi^+(K^+\pi^-\pi^-)$ and the mesons are produced with $\gamma \approx 5$. (Describe the necessary features of the experiment and how the measurement would be performed)

Elementary Particle Physics 2

- (a) (5 points) How much of the mass of the Universe is due to dark energy, dark matter and visible matter? How does one detect the presence of dark matter and how can one estimate how much dark energy there is?
- (b) (5 points) The anomalous magnetic moment of the muon gets contributions from the strong, weak and electromagnetic interactions. Write down a Feynman diagram that contributes to each of these cases in lowest order of loops.
- (c) (5 points) The electromagnetic, weak and strong coupling constants are dimensionless. As functions of energy, they seem to come together at some high energy. Which of them is rising as a function of energy, and which is falling? The Newton coupling constant is dimensionful. What is the implication for the effective coupling constant of gravity as a function of energy?
- (d) (5 points) The Higgs mechanism gives a mass to some of the particles of the Standard Model. Suppose one deletes the Higgs fields and their mass and interactions from the Standard Model, but one retains the mass terms of the other particles. Which of the following statements would follow? Give a brief explanation of each answer.
 - (i) unitarity is violated,
 - (ii) renormalizability is violated,
 - (iii) the weak mixing angle vanishes.

Atomic, Molecular, and Optical Physics 1

This problem asks about the quantum mechanical description of the atoms with a single electron outside otherwise closed electron shells.

- (a) (7 points) In 1913 Bohr used the classical force balance for circular motion, $mv^2/r = e^2/r^2$, to derive the energies for such “single electron” atoms. He found $E_n = -m_0c^2\alpha^2/2n^2$, where m_0 is the electron rest mass, $\alpha = e^2/\hbar c \approx 1/137 \approx 7.3 \times 10^{-3}$ is the Sommerfeld fine structure constant, and $n \geq 1$ is an integer. Contrary to most elementary texts, Bohr did NOT quantize angular momentum and he knew nothing of de Broglie waves in 1913. Instead, he assumed that transitions between adjacent very large orbits, $n \gg 1$, with $\Delta n = \pm 1$, had to be accompanied by radiation at the orbital frequency v/r . Show that his procedure is consistent with quantizing the angular momentum as $mvr = n\hbar$.
- (b) (7 points) It is possible to calculate the relativistic corrections to the energies arising from the motion of the electron in accordance with Einstein’s formula, $E^2 = p^2c^2 + m_0^2c^4$. Expand this expression for the energy and compare the magnitude of the relativistic correction with Bohr’s value E_n given above.
- (c) (6 points) In Bohr’s formula, the energies depend only on a single quantum number, n . However, in many multi-electron atoms this is not the case, and transitions between states of the same n lead to distinct spectral lines. Give a classical-physics explanation of why this is so (simply giving the name, quantum defect, is NOT sufficient).

Atomic, Molecular, and Optical Physics 2

This problem addresses some basic physics of ultracold atomic Bose-Einstein condensates (BECs).

- (a) (4 points) Consider an ideal gas of atoms (gas density n , atomic mass m) at temperature T . Derive an expression for the typical de Broglie wavelength of atoms in the gas.
- (b) (4 points) There are *two* typical length scales in the gas. What is the other one? For quantum statistics to become important, what must the relationship between the two length scales be? From this relationship, derive an expression for the corresponding temperature, and calculate it for a gas of ^{87}Rb atoms at density $n = 5 \times 10^{12} \text{ cm}^{-3}$ ($m = 1.4 \times 10^{-25} \text{ kg}$).

At zero-temperature, the ground state wavefunction $\Psi(\vec{r})$ of a Bose-Einstein condensate containing N *weakly interacting* atoms can be written as a $\Psi = \sqrt{N}\psi$, where $\psi(\vec{r})$ is an (effective) single particle wavefunction that is subject to a *mean field* potential $V_i = g|\psi|^2$, with g a positive constant. Suppose now that this BEC is confined in a harmonic trap with potential $V_{tr}(\vec{r}) = \frac{1}{2}m(\omega r)^2$, where ω is the trap frequency.

- (c) (4 points) Write down the Schrödinger equation for ψ (denote the energy eigenvalue, also known as the chemical potential, by μ), and solve it under the assumption that the kinetic term can be neglected. What is the functional form of the atomic density in the trap?
- (d) (4 points) Derive an expression for the width of ψ , and calculate it for $\omega = 2\pi \times 50 \text{ Hz}$ and $\mu = k_B \times 200 \text{ nK}$.
- (e) (4 points) The mean-field potential strongly affects the size of the BEC in the trap. Derive an expression for the width of the single particle harmonic oscillator ground state in the trapping potential. For the the given parameters ($\omega = 2\pi \times 50 \text{ Hz}$, $\mu = k_B \times 200 \text{ nK}$), what is the ratio of the width of the BEC and the width of the harmonic-oscillator ground state?

Condensed Matter Physics 1

The screening of an impurity charge in a three-dimensional free-electron gas is described by the famous Lindhard dielectric function. Consider a simplified approximate form for this dielectric function

$$\epsilon(q) = 1 + \frac{k_0^2}{q^2} L(q/2k_F), \quad \text{where} \quad L(x) = \begin{cases} 1 & \text{for } x < 1 \\ 0 & \text{for } x > 1 \end{cases}$$

and the Thomas-Fermi wavevector, k_0 , and the Fermi wavevector, k_F , are constants. Investigate the asymptotic behavior of the electric potential $\phi(r)$ around a point charge Q .

- (a) (6 points) Calculate $\phi_{\text{bare}}(q)$, the Fourier transform of the bare (unscreened) potential.
- (b) (6 points) Use the above form for the dielectric function to show that the screened potential takes the form

$$\phi(r) = \frac{2Q}{\pi r} \int_0^\infty F(q) \sin(qr) dq ,$$

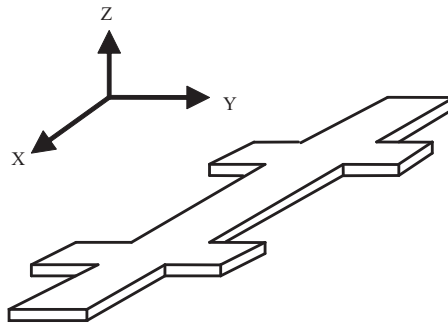
identify $F(q)$, and find its discontinuity, ΔF , at $q = 2k_F$.

- (c) (6 points) Make use of the discontinuity to calculate the r -dependence of $\phi(r)$ at long distances.
- (d) (2 points) Provide an intuitive (qualitative) explanation for your result, known as a Friedel oscillation.

Condensed Matter Physics 2

In what follows, show how you would determine some of the microscopic characteristics of an unknown conducting material by performing simple electrical measurements and interpreting them through simple theoretical models. Explain everything you will do in sufficient detail to demonstrate your understanding. For each measurement show placement of the leads on the sample, and a circuit diagram.

The material sample is a thin layer patterned into a double-cross shape as shown. Using a thickness profiler and calibrated optical microscope you can measure any of the sample's geometrical parameters (thickness t , width w , and various distances). Further, you can place electrical leads (contacts) on this sample at the points of your choosing. In your lab, you have standard resistors in a wide range of values, a battery, several voltmeters, several ammeters, and a calibrated electromagnet which can produce a uniform magnetic field.



- (5 points) Making a simple relaxation-time approximation, present an elementary theory of electrical resistivity and derive Ohm's law. Express the resistivity, ρ , in terms of microscopic parameters. Discuss which microscopic parameters, if any, you will be able to determine once you measure ρ . *A priori* you know only the magnitude of the carrier charge, which is equal to electron charge e .
- (5 points) Experimentally, you actually measure a resistance R (ratio of a voltage to a current). Discuss and explain the experimental arrangement which will exclude the influence of contact and lead resistances on your measurement of ρ , and give the relationship between ρ and R .
- (7 points) Present an elementary theory of the classical Hall effect, assuming that there is only one type of charge carrier. Show how to perform such a measurement (place contacts, show circuit). Explain how this measurement would be used to determine the carrier density, n , of the sample.
- (3 points) Assuming that the carriers in your sample are free electrons, with the charge, e , and mass, m , of an electron, what other microscopic parameter in the formula for ρ can you determine by combining the results of (a) and (c)? Explain how.

Nuclear Physics 1.

Consider a model nucleus ${}^8_8\text{O}$ which has 8 neutrons and 8 protons, all confined to the lowest three shell model orbits.

- (a) (4 points) What is the root-mean-square radius of this nucleus?
- (b) (4 points) What are the total spin J and total isospin T of this nucleus? Explain how your results are obtained.

Consider now two other model nuclei X and Y: X has two neutrons confined in the shell-model orbit $0d_{5/2}$ and Y has one proton and one neutron confined in the same orbit. The total angular momentum and isospin for X and Y are respectively (J_X, T_X) and (J_Y, T_Y) .

- (c) (4 points) What are the allowed values for (J_X, T_X) and (J_Y, T_Y) ?

The transition operator for β -decay is given by $[G_V \sum_{j=1}^A \tau_{\pm}(j) + G_A \sum_{j=1}^A \sigma(j) \tau_{\pm}(j)]$.

- (d) (4 points) From what states of X to what states of Y are β -decay of the Fermi (vector) type allowed?
- (e) (4 points) Answer the same question for the β -decay of the Gamow-Teller (axial) type.

Nuclear Physics 2.

You will use carbon dating to determine the age of an old sample of cloth. Living material maintains a concentration of ${}^{14}\text{C}/{}^{12}\text{C} = 1.2 \times 10^{-12}$. Once dead, the ${}^{14}\text{C}$ β -decays with a half-life of 5730 years. Assume that the cloth is roughly 2000 years old and that your sample contains 1 milligram of carbon.

- (a) (2 points) What are the decay products of ${}^{14}\text{C}$?
- (b) (3 points) What precision in the measurement of ${}^{14}\text{C}/{}^{12}\text{C}$ is required to measure the sample's age to an accuracy of 50 years? Express your result as a percentage.
- (c) (4 points) If you want to measure the amount of ${}^{14}\text{C}$ by measuring its radioactive decay, and assuming that you could detect 100% of all decays into 4π , how long would a counting experiment run to produce the required precision?

Wisely, you choose Accelerator Mass Spectrometry (AMS) using a Tandem Van de Graaff accelerator. Carbon ions enter the tandem in charge state -1 and convert to charge state +5. The terminal voltage of the accelerator is +7MV.

- (d) (4 points) Determine the magnetic field required to bend these ions with a radius of one meter.
- (e) (4 points) If the ${}^{12}\text{C}$ beam current exiting the tandem is $1 \mu\text{A}$, calculate the time required to make a measurement with 50 year precision.
- (f) (3 points) Explain qualitatively why each of the following items are necessary for AMS:
 - (i) Beam energy larger than 1 MeV.
 - (ii) Charge state larger than +3.
 - (iii) Magnetic and Electric fields.

Astronomy 1

The James Webb Space Telescope (JWST) is a large infrared telescope with a 6.5 m diameter primary mirror scheduled to be launched in 2013. According to NASA,

JWST will observe primarily the infrared light from faint and very distant objects. But all objects, including telescopes, also emit infrared light. To avoid swamping the very faint astronomical signals with radiation from the telescope, the telescope and its instruments must be very cold. Therefore, JWST has a large shield that blocks the light from the Sun, Earth, and Moon, which otherwise would heat up the telescope, and interfere with the observations. To have this work, JWST must be in an orbit where all three of these objects are in about the same direction.

- (a) (4 points) Why is putting the telescope in space worth the considerable effort and expense?
- (b) (4 points) The Mid-Infrared Instrument (MIRI) will observe light of $3 - 8\mu\text{m}$. What is the limiting angular resolution that can be observed by the JWST with light in this range?
- (c) (4 points) The plan is to locate JWST in an orbit around one of the Lagrange points of the Earth's orbit. (A Lagrange point is one of the positions in the orbital configuration of the Sun and Earth where a small object affected only by gravity can theoretically orbit at the same period as the Earth.) Sketch the Lagrange points for the Earth's orbit. Indicate the Lagrange point at which the Earth and Sun are in about the same direction. (It turns out that this is outside of the Moon's orbit.)
- (d) (4 points) Suppose that the heat shield is in the form of a disk with a diameter of 10 meters. The side facing the sun is made with a Bond Albedo (the fraction of solar radiation scattered) of 0.99, whereas the side facing the satellite is black. If the telescope orbits the Sun at 1 AU, what will be the steady state temperature of the heat shield? (Solar luminosity $L_{\odot} = 3.826 \times 10^{26}$ W, 1 AU = 1.496×10^{11} meter, and the Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ J s⁻¹ m⁻² K⁻⁴)
- (e) (4 points) How could the JWST be passively cooled to a lower temperature than your answer for (d)?

Astronomy 2

- (a) (10 points) Estimate the temperature and density at the Sun's center, using a model that predicts

$$\rho(r) = \rho_c \left[1 - \frac{r^2}{R^2} \right]$$

where ρ_c is the central density and R is the Sun's radius. Assume the equation of state is that of a perfect ionized gas with a composition of 75% H and 25% He by mass. Compared to the actual values at the Sun's center, how do these compare? (Solar mass $M = 2 \times 10^{33}$ g, radius $R = 7 \times 10^{10}$ cm.)

- (b) (10 points) Demonstrate for a star fusing H to He with the p-p process that the Pop II main sequence is shifted below and to the left of the Pop I main sequence in the Hertzsprung-Russell diagram. For a given mass, also show that a Pop II star has a larger effective temperature T_{eff} and a shorter lifetime than a Pop I star.

Astronomy 3

Measurement of the one-dimensional incidence per line of sight $n(z)$ of high-redshift “damped Lyman alpha” absorption systems in the spectra of background QSOs can be used to determine the cosmological mass density of neutral gas at early epochs, where $n(z)dz$ is the mean number of absorption systems intercepted per line of sight in the redshift interval dz around redshift z . Consider a class of damped Lyman alpha absorption systems of projected or “column” density $N = 10^{21} \text{ cm}^{-2}$ of neutral Hydrogen measured at redshift $z = 3$. Take the one-dimensional incidence per line of sight to be $n(z) = 0.1$, and consider an Einstein–de Sitter cosmological model, for which the relationship between cosmic time t and redshift is $t = \frac{2}{3H_0}(1+z)^{-3/2}$, where H_0 is the Hubble constant. Take the Hubble constant to be $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which implies a Hubble time $\frac{1}{H_0} = 14 \times 10^9 \text{ yr}$ and a Hubble length $\frac{c}{H_0} = 4300 \text{ Mpc}$, where c is the speed of light. The proton mass is $m_p = 1.7 \times 10^{-24} \text{ g}$, the solar mass is $M_\odot = 2.0 \times 10^{33} \text{ g}$, and $1 \text{ pc} = 3.1 \times 10^{24} \text{ cm}$.

- (a) (4 points) What is the age of the Universe at the redshift $z = 3$ of the measurement?
- (b) (8 points) What is the *proper* mass density ρ_{prop} traced by the absorption systems, in units of $M_\odot \text{ Mpc}^{-3}$?
- (c) (4 points) What is the *comoving* mass density ρ_{co} traced by the absorption systems, in units of $M_\odot \text{ Mpc}^{-3}$?
- (d) . (4 points) Compare your answer to part b or c, as appropriate, to the mass density of stars in galaxies in the local Universe, which is roughly $1.4 \times 10^8 M_\odot \text{ Mpc}^{-3}$. What conclusion might be drawn from this comparison?

Astronomy 4

One of the essential results of the theory of stellar interiors is the *Mass-Luminosity* relation which gives the dependence of a star’s luminosity, L , on its mass, M , while the star is on the main sequence.

- (a) (5 points) What *is* the main sequence?
- (b) (5 points) What are the parameters of the star that must be measured in order determine the star’s luminosity? Which parameter is usually the most difficult to determine correctly?

The only reliable way to measure stellar masses is gravitational, by studying the orbital motion of stars in a double star system.

- (c) (5 points) What are the parameters that must be determined in order to measure the total mass, $M_1 + M_2$, of the stars in the double star system?
- (d) (5 points) Given a successful measurement of the parameters in (c), what additional parameters must be measured in order to determine the individual masses, M_1 and M_2 ?